

Lately published by I. Jackson, in Meath-street.

**T**HE *Schoolmaster's Assistant*; being a Compendium of Arithmetic, in four Parts. Containing, 1st, Arithmetic in whole Numbers. 2d, Vulgar Fractions. 3, Decimals, Extraction of Roots, Interest, &c. 4th, A large Collection of Questions for the Exercise of Learners. The Whole recommended by several eminent Mathematicians and Schoolmasters, as necessary to be used in Schools by all Teachers, who would have their Scholars thoroughly understand and make a quick Progress in Arithmetic. By *Tho. Dilworth*, Schoolmaster in *Wapping*. To which is added, a very necessary and useful Appendix, containing a particular Account of the Coins, Weights and Measures used in *Ireland*. Price 1s. 8d.

*Volter Eliminated* of the Rules of Arithmetic Digested and Explained in a Methodical Course of Questions and Examples, with a Copper plate Title. Price 2s. 2d.

The *Elements of Euclid*; with Select Theorems out of Archimedes, by the learned *Andrew Tacquet*. To which are added, Practical Corollaries, shewing the Uses of many of the Propositions, by *William Whiston*, M. A. Mr. *Lucas's* Professor of Mathematicks in the University of Cambridge.

--- In this Sixth Edition is added an Appendix of Practical Geometry, with Forty new Figures, and a Brief independent Demonstration of certain select and most useful Propositions. By *S. F.* Price 4s. 4d.

A new Edition of *Harvey's Mathematical Measuring*, with sundry Corrections and Additions 2s. 8d. half.

*De Moivre* on Annuities and Interest. 2s. 8d. halfp.

*Hill's Arithmetic*, the new improved Edition, with *Albby's* Solution of the 100 Questions 5s. 5d.

*Apby's Algebra*, or young Analyst's Exercise. 2s. 2d.

Practical Arithmetick, Extracted from the larger Treatise encouraged by Subscription, and adapted to the Commerce of both Great Britain and Ireland, with an elegant Copper plate Title. By *John Gough*, Teacher, and Author of the Practical English Grammar &c. neatly bound price 2s. 8d. half, in Calf 3s.

*Talbot's Arithmetical Catechism*, 1s.

*Inspector or Young Man's Companion*, much more correct than any other Edition. Price 1s. 1d.

*Art's Master Piece*, or *Companion* for the Ingenious, especially the Female Sex; in 2 parts, 1st, the Art of Limning or Painting in Oil &c. 2d, the Art of making Glass of Crystal &c. with many other Curiosities. Price 1s.

*Art's Companion* in three Parts, or an Assistant for the Ingenious; 1st, in Drawing and Painting. 2d, in Varnishing. 3d, in Japanning, Casting, making Colours, &c. Price 1s. 8d.



Ingenious COCKER, now to Rest thou'rt gone  
No *Art* can shew thee fully, but thine own :  
Thy rare *Arithmetick*, alone can show,  
Th'vast *Sums* of *Thanks* we for thy *Labours* owe



# COCKER'S ARITHMETIC:

BEING

A plain and familiar Method, suitable to the meanest Capacity, for the full understanding of that incomparable ART, as it is now taught by the ablest Schoolmasters in CITY and COUNTRY.

COMPOSED

By EDWARD COCKER, late Practitioner in the Arts of Writing, Arithmetick and Engraving; being that so long since promised to the World.

Perused and published

By JOHN HAWKINS, Writing Master, near St. George's Church in *Southwark*, by the Author's correct Copy, and commended to the World by many eminent Mathematicians and Writing-Masters in and near *London*.

---

The Fifty first EDITION, carefully corrected and amended. With Notes upon the *high* Weights and Measures, &c.

By  
ISAAC JACKSON, and Assistants.

---

Licensed Sept. 3, 1677. Roger L'Estrange.

---

D U B L I N :

Printed by and for I. JACKSON and Son, at the  
*Globe* in *Aleath-street*. 1789.

TO his much honoured Friends, *Manwaring Davis*  
of the *Inner Temple*, Esq; and *Mr. Humphry Davis*  
of *St. Mary Newington Butts*, in the County of *Surry*,  
*John Hawkins*, as an Acknowledgment of unmerited  
Favours, humbly dedicateth this *Manual of Arithmetick*.

To the READER.

*Courteous Reader,*

I Having had the Happiness of an intimate Acquaintance with *Mr. Cocker* in his Life-time, often solicited him to remember his Promise to the World, of publishing his *Arithmetick*; but (for Reasons best known to himself) he refused it; and after his Death (the Copy falling accidentally into my Hand) I thought it not convenient to smother a Work of so considerable a Moment, not questioning but it might be as kindly accepted as if it had been presented by his own Hand. The Method is familiar and easy, discovering as well the Theoric as the Practic of that necessary Art of *Vulgar Arithmetick*. And in this new Edition there are many remarkable Alterations for the Benefit of the Teacher or Learner, which I hope will be very acceptable to the World. I have also performed my Promise, in publishing the *Decimal Arithmetick*, which finds Encouragement to my Expectations, and the Booksellers too. I am thine to serve thee,

*John Hawkins.*

AT the Request of *Mr. Jackson*, I have examined this new Edition of *Cocker's Arithmetic*: I believe scarce any of the Errors complained of in the former Impressions have passed without Amendment, and the Learner may now depend on its Correctness.

May 31,  
1757.

*G. Minty*

# Mr. Edward Cocker's

## PROEM or PREFACE.

**B***Y the secret Influence of Divine Providence, I have been instrumental to the Benefit of many, by Virtue of these useful Arts, Writing and Engraving : And do now with the same wonted Alacrity, cast this my Arithmetical Wite in'o the publick Treasury, beseeching the Almighty to grant the like Blessing to these as to my former Labours.*

Seven Sciences supremely excellent,  
Are the chief Stars in Wisdom's Firmament :  
Whereof *Arithmetick* is one, whose Worth  
The Beams of Profit and Delight shine forth ;  
This crowns the rest, this makes Man's Mind compleat,  
This treats of Numbers, and of this we treat.

*I have been often desired, by my intimate Friends, to publish something on this Subject, who, in a pleasing Freedom, have signified to me, that they expected it would be extraordinary. How far I have answered their Expectation I know not ; but this I know, that I have designed this Work not extraordinary abstruse or profound, but have, by all Means possible, within the Circumference of my Capacity, endeavoured to render it extraordinary useful to all those, whose Occasions shall induce them to make use of Numbers. If it be objected, That the Books already published, treating of Numbers, are innumerable ; I answer, That's but a small Wonder, since the Art is infinite. But that there should be so many excellent Tracts of Practical Arithmetick extant, and so little practised, is to me a great Wonder ; knowing, that as Merchandize is the Life of the Weal publick, so Practical Arithmetick is the Soul of Merchandize. Therefore I do ingenuously profess, that in the Beginning of this Undertaking, the numerous Concerns of the honoured*

A 3

*Merchant*

*Merchant first possesseth my Consideration : And how far I have accommodated this Composure for his most worthy Service, let his own profitable Experience judge.*

Secondly, *For your Service, most excellent Professors, whose Understandings soar to the Sublimity of the Theory and Practice of this noble Science, was this Arithmetical Tractate composed ; which you may please to employ as a Monitor to instruct your young Tyroes, and thereby take Occasion to reserve your precious Moments, which might be exhausted that Way, for your more important Affairs.*

Thirdly, *For you the ingenious Offspring of happy Parents, who will willingly pay the full Price of Industry and Exercise for those Arts and choice Accomplishments, which may contribute to the Felicity of your future State : For you, I say, (ingenious Practitioners) was this Work composed, which may prove the Pleasure of your Youth, and the Glory of your Age.*

Lastly, *For you the pretended Numerists of this vapouring Age, who are more disingeniously witty to propound unnecessary Questions, than ingenuously judicious to resolve such as are necessary ; for you was this Book composed and published, if you will deny yourselves so much as not to invert the Streams of your Ingenuity, but by studiously conferring with the Notes, Names Orders, Progrejs, Species, Properties, Proprieties, Proportions, Powers, Affections and Applications of Numbers delivered herein, become such Artists indeed as you now only seem to be, This Arithmetick, ingeniously observed and diligently practised, will turn to good account to all that shall be concerned in Accompts, since all its Rules are grounded on Verity, and delivered with Sincerity ; the Examples built up gradually from the smallest Consideration to the greatest ; and all the Problems or Propositions well weighed, pertinent and clear, and not one of them throughout the Tract taken upon Trust, therefore, now,*

*Zolius and Momus. lie you down and die.  
For these Inventions your whole Force defy.*

*Edward Cocker.*





*Courteous Reader,*

**B**Eing well acquainted with the deceased Author, and finding him knowing and studious in the Mysteries of Numbers and Algebra, of which he had some choice Manuscripts, and a great Collection of printed Authors in several Languages, I doubt not but he hath wit his Arithmetick suitable to his own Preface, and worthy Acceptation, which I thought fit to certify, on a Request to that Purpose, made to him that wisheth thy Welfare, and the Progress of Arts.

Nov. 27.  
1677.

*John Collens.*

---

*This Manual of Arithmetick is recommended to the World by us whose Names are subscribed, viz.*

Mr. John Collens	} <i>Math.</i>	Mr. William Mason
Mr. James Atkinson		Mr. Stephen Thomas
Mr. Peter Perkins		Mr. Peter Storey
Mr. Rich. Lawrance, Sen.		Mr. Benj. Tichbourn
Mr. Eleazar Wigan		Mr. Joseph Symmonds
Mr. Rich. Noble of Gilford		Mr. Jerem. Miles
Mr. William Norgate		Mr. Josiah Cuffley
		Mr. John Hawkins

*And generally approved by all ingenious Artists.*

# A TABLE of the CONTENTS of this Book.

	Chap.
<b>N</b> otation of Numbers	1
Of the natural Division of Integers, and the Denomination of their Parts	2
Of the Species or Kinds of Arithmetick	3
Of the Addition of whole Numbers	4
Of Subtraction of whole Numbers	5
Of Multiplication of whole Numbers	6
Of Division of whole Numbers	7
Of Reduction	8
Of Comparative Arithmetick, viz, the Relation of Numbers one to another	9
The single Rule of Three direct	10
The single Rule of Three inverse	11
The double Rule of Three direct	12
The double Rule of Three inverse	13
The Rule of Three composed of five Numbers	14
Single Fellowship	15
Double Fellowship	16
Alligation medial	17
Alligation Alternate	18
Reduction of Vulgar Fractions	19
Addition of Vulgar Fractions	20
Subtraction of Vulgar Fractions	21
Multiplication of Vulgar Fractions	22
Division of Vulgar Fractions	23
The Rule of Three direct in Vulgar Fractions	24
The Rule of Three inverse in Vulgar Fractions	25
Rules of Practice	26
The Rule of Barter	27
Questions in Loss and Gain	28
Equation of Payments	29
Exchange	30
Single Position	31
Double Position	32



## C H A P. I.

*Notation of Numbers.*

**A**RITHMETICK is an Art of Numbering, or Knowledge, which teacheth to number well. And there are divers Species and Kinds of *Arithmetick* and *Geometry*, the which we do intend to treat of in Order, applying the Principles of the one to the Definition of the other. For as Greatness is the Subject of *Geometry*, so Number is the Subject of *Arithmetick*; and if so, then their first Principles and chief Fundamentals must have like Definitions, or at least some Congruency.

2. Number is that by which the Quantity of any Thing is expressed or numbered; as the Unit is the Number by which the Quantity of one Thing is expressed or said to be one, and two, by which it is named two, and  $\frac{1}{2}$  half, by which it is named or call half; and  $\sqrt{3}$  the Root of 3, by which it is call the Root of 3; the like of any other.

3. Hence it is that Unit is Number; for the Part is of the same Matter that is its Whole, the Unit is part of the Multitude of Units, therefore the Unit is of the same Matter, that is the Multitude of Units; but the Matter of the Multitude of Units is Number; therefore the Matter of Unit is Number; or else, if from a Number given no Number be subtracted, the Number given remaineth; as suppose 3 the given Number; if, as some suppose, 1 to be no number, then if you subtract 1 from 3, there must remain three still; which is very absurd.

4. Hence it will be convenient to examine from whence Number hath its Rise or Beginning. Most Authors maintain, that Unit is the Beginning of Number, and itself no Number; but looking upon the Principles and Definitions in the first Rudiments of *Geometry*, we shall find that the Definition of a Point is no way congruous with the Definition of an Unit in *Arithmetick*; and therefore One or Unit must be in the Bounds or Limits of Number, and consequently the Beginning of Number is not to be found in the Number 1; wherefore making Number and Magnitude congruent in Principles, and like in Definitions, we make

and constitute a Cypher to be the Beginning of Number, or rather the Medium between encreasing and decreasing Numbers, commonly called absolute or whole Numbers, and negative and fractional Numbers, between *which* nothing can be imagined more agreeable to the Definition of a Point in Geometry; for as a Point is an Adjunct of a Line, and itself no Line, so is a (o) Cypher an Adjunct of Number, and itself no Number: And as a Point in Geometry cannot be divided or increased into Parts, so likewise (o) cannot be divided or increased into Parts; for as many Points, tho' in Number infinite, do make no Line,

so many (o) Cyphers, tho' in Number infinite, do make no Number. For the Line AB cannot be increased by the Addition of the Point

A-----B C

D 6 C, neither the Number D be increased by the

E o Addition of the (o) Cypher E; for if you add

--- nothing to 6. the Sum will be 6, (o) Cypher

Sum of 6 neither increasing nor diminishing the Num-

ber 6; but if it be granted that AB be ex-

A--B--C tended or prolonged to the Point C, so that

AC be made a continued Line, then AB is

DE 6 increased by the Addition of the Point C. In

6o like manner, if we grant D (6) to be pro-

longed to E (o), so that DE (6o) be a con-

tinued Number, making 6o, then (6) is augmented by

the Aid of (o) as constituting the Number (6o) Sixty:

And furthermore, that 1 or Unit is material, in a Num-

ber, and that (o) is the Beginning of Number, is proved

by all Authors, altho' indirectly; for the Tables of Sines

and Tangents prove one Degree to be a Number, because

the Sine of 1 Degree is 174524. (the Radius being 1000000)

and the Beginning of the Table is (o), and it answereth

ooooo, &c.

5. Hence it is that Number is not Quantity discontinu'd

for that which is but one Quantity, is not Quantity dis-

junct: 6o Sixty, as it is a Number, is one Quantity, viz.

one Number (6o) sixty; therefore as it is a Number, it is

not Quantity disjunct, for Number is some such Thing in

Magnitude; as Humidity in Water; for as Humidity ex-

tends itself thro' all and every part of Water, to Number

related to Magnitude doth extend itself thro' all and every

part of Magnitude: Also, as continued Water doth answer

continued Humidity, so to a continued Magnitude doth

answer a continued Number. As the continued Humidity

of an entire Water suffereth the same Division and Distinc-

tion that his Water doth, so the continued Number suffer

eth



eth the same Division and Distinction that his magnitude doth. And thus much concerning the Definition and Principles of Number and Magnitude. We come now to treat of,

6. The Characters or Notes by which Numbers are signified, or by which a Number is ordinarily expressed; and they are these, *viz.* (0) Cypher or Nothing, 1 One, 2 Two, 3 Three, 4 Four, 5 Five, 6 Six, 7 Seven, 8 Eight, 9 Nine. The Cypher, which tho' of itself it expresseth not any certain or known Quantity, yet is the Beginning or Root of Number, and the other nine Figures are called significant Figures of Digits.

7. In Number of any Sort two Things are to be considered, *viz.* *Notation* and *Numeration*.

8. *Notation* teacheth how to describe any Number by certain Notes and Characters, and to declare the Value thereof, being so described, that is by Degrees and Periods.

9. A Degree consists of 3 Figures, *viz.* of three Places, comprehending Units, Tens and Hundreds, so 365 is a Degree, and the first Figure (5) on the right-Hand, stands simply for its own Value, being Units, or so many Ones, *viz.* five; the second in Order from the Right, signifies as many times Ten as there are Units contained in it, *viz.* sixty; the third in the same Order signifies so many Hundreds as it contains Units, so will the Expression of the Number be Three hundred sixty five, &c.

10. A Period is when a Number consists of more than 3 Figures or Places, and whose proper Order is to prick every third Place, beginning at the right Hand, and so on to the left; so the Number 63,452 being given, it will be distinguished thus, 63,452, and expressed thus, sixty three thousand four hundred fifty two; likewise 4,578,236,782, being distinguished as you see, will be expressed thus, four thousand five hundred seventy eight millions, two hundred thirty six thousand, seven hundred eighty two.

11. Number is either Absolute or Negative.

12. Absolute, intire, whole, increasing Number, is that by which annexing another Figure or Cypher, it becomes ten times as much as it stood for before; and if two Figures or Cyphers be annexed, it makes an hundred times as much as it stood for before, &c. as if you annex to the Figure 6 a Cypher, then it will be (60) sixty; so if two Cyphers are annexed, then it will be (600) six hundred, and if you do annex to it (4) four, then it will be (64) sixty four, and if you annex (78) seventy eight, it will be then (678) six hundred seventy eight. &c.

13. A negative or broken, fractional, decreasing Number, is that by which prefixing a Point or Prick toward the left Hand, its Value has decreased from so many Units to so many tenth Parts of any Thing; and if a Point and (o) Cypher, or Digit, be prefixed, it will be then so many hundred Parts; and if a Point and two Cyphers or Digits be prefixed, its Value is decreased to be so many thousandth Parts; as if you would prefix before the Figure 3 a Point (.) or Prick thus (.3) it is then decreased from 3 Units or 3 Integers, to 3 tenth Parts of an Unit or an Integer; and if you prefix a Point and Cypher thus (.03) it is decreased from 3 Integers to 3 hundred Parts of an Integer; and by this Means  $\frac{5}{10}$  absolute, by prefixing of a Point, will be decreased to  $\frac{5}{100}$  negative, which is 5 tenth Parts of a Pound, equal in value to ten Shillings, and so by prefixing of more Cyphers or Digits, its Value is decreased in a decuple Proportion *an infinitum*. As in the following Scheme, or rather Order of Numbers, we have placed (o) Cypher in its due Place in Order, as it is in the Beginning and Medium of Number; for going from (o) towards the left Hand, you deal with intire, absolute, whole, increasing Numbers

Increasing Numbers.				Decreasing Numbers.				
25	629	876	543	21012	345	678	976	3
mm	mm	mm	mm	CXUXC	mm	mm	mm	m
mm	mm	mm	Cx		XC	mm	mm	m
mm	mm	CX				XC	mm	m
mm	CX						XC	m
X								M

But going from (o) the Place of Units towards the right Hand, you meet with broken, negative, fractional and decreasing Numbers. And hence it follows, that *Multiplication* increaseth the Product in absolute Numbers, but decreaseth the Product in negative Numbers; also *Division* decreaseth the Quotient in whole Numbers, and increaseth it in negative fractional Numbers.

14. An absolute intire, whole, increasing Number, always has a Point annexed towards the right Hand; and therefore,

15. A negative, broken, decimal, decreasing Number, hath always a Point prefix'd towards the left Hand. When we express Integers or whole Numbers, as 5 Pounds, 5 Feet, 26 Men, we usually annex a Point or Prick after the Number, thus,

l. feet, men. inch.  
5. 5. 26. 347.  
But

But when we express Decimals, or Numbers that are denied to be intire, or decreasing Number, we do commonly prefix a Point or Prick before the said Decimal or decreasing Number, thus (.3) that is, 3 Tenths, or 3 Primes (.03) that is 3 Hundredths, or 3 Seconds.

16. A whole or absolute Number is a Unit, or a composed Multitude of Units, and it is either a Prime or else a compound Number.

17. Prime Numbers amongst themselves, are those which have no Multitude of Units for a common Measure, as 8 and 7, or 10 and 13, because not any Multitude of Units can equally measure or divide them without a Remainder.

18. Compound Numbers amongst themselves are those which have a Multitude of Units for a common Measure, as 9 and 12, because 3 measures them exactly, and abbreviates them to three and four.

19. A broken Number, commonly called a Fraction, is a Part or Parts of a whole Number, viz. A Part of an Integer, as  $\frac{1}{3}$  one Third, is one third Part of an Unit.

20. A broken Number or Fraction consists of two Parts, viz. the Numerator and Denominator.

21. The Numerator and Denominator of a Fraction are set one over the other, with a Line between them; and the Numerator is set above the Line, and expresseth the Parts therein contained.

22. The Denominator of a Fraction, is the inferior Number placed below the Line, and expresseth the Number of Parts, into which the Unit of Integer is divided; and let  $\frac{3}{4}$  be the Fraction given, so shall 3 be the Numerator, and doth express or number the Multitude of Parts contained in this Fraction; for  $\frac{3}{4}$  is a Fraction compounded of Fourths or Quarters, and the Figure 3 in numbering shews us, that in that Fraction there are 3 of the 4 Parts or Quarters, also in the same Fractions, 4 is the Denominator, and doth express the Quality of the Fraction, viz. that the Whole or Integer is divided into 4 equal Parts.

23. A broken Number is either proper or improper, viz. proper when the Numerator is less than the Denominator, for  $\frac{3}{4}$  is a perfect proper Fraction, but an improper Fraction hath its Numerator greater, or at least equal to the Denominator, thus  $\frac{5}{4}$  is an improper Fraction, the Reason is given in the Definition.

24. A proper broken Number is either simple or compound, *viz.* simple when it hath one Denomination, and compound when it consisteth of divers Denominations; if  $\frac{3}{4}$ ,  $\frac{5}{12}$ ,  $\frac{25}{100}$  were given, we say they are each of them single or simple Fractions, because they consist but of one Numerator and one Denominator; but if  $\frac{3}{4}$  of  $\frac{6}{12}$  of  $\frac{25}{100}$  of a Pound sterling were given, we say that it is a compound broken Number or Fraction, because the Expression and Representation consisteth of more Denominations than one, and such by some are called Fractions of Fractions; they have always this Particle (of) between them.

25. When a single broken Number or Fraction hath for his Denominator a Number consisting of a Unit in the first Place towards the left Hand, and nothing but Cyphers from the Unit towards the right Hand, it is then the more aptly and rightly called a Decimal Fraction; under this Head are all our decreasing Numbers placed, and in our 13th Definition, called Negative; and by the Order there prescribed, we order them to be Decimals, by signing a Prick or Point before them, or the Numerator, rejecting the Denominator; therefore according to our last Rule,  $\frac{5}{10}$ ,  $\frac{5}{100}$ ,  $\frac{25}{1000}$ , are then said to be decimals; and a Decimal Fraction may be expressed without its Denominator (as before) by prefixing a Point or Prick before the Numerator of the said Fraction and then shall the former Fractions  $\frac{5}{10}$  and  $\frac{25}{1000}$  stand thus, .5. and .025.

But oftentimes, as in the second and fourth Fractions,  $\frac{5}{100}$  and  $\frac{25}{1000}$ , a Prick or Point will not do without the Help of a Cypher or Cyphers prefixed before the significant Figures of the Numerator, and therefore when the Numerator of a Decimal Fraction consisteth not of so many Places as the Denominator hath Cyphers, fill up the void Places of the Numerator with prefixing Cyphers before the significant Figures of the Numerator, and then sign it for a Decimal, so shall  $\frac{5}{100}$  be .05, and  $\frac{25}{1000}$  will be .025, and  $\frac{72}{10000}$  will be .0072. Now by this we may easily discover the Denominator; having the Numerator, for always the Denominator of any Decimal Fraction consists of so many Cyphers as the Numerator hath



ath Places, with an Unit prefixed before the said Cyphers, *viz.* under the Point or Prick.

26. A Decimal Number or Fraction, is expressed by Primes, Seconds, Thirds, Fourths, &c. and its Number decreasing. Here instead of natural and common Fractions, as  $\frac{3}{4}$  of a Thing, we order the Thing or Integer into Primes, Seconds, Thirds, Fourths, Fifths, &c. that our Expression may be consonant to our former Order.

27. In Decimal Arithmetick we always imagine that all intire Units, Integers and Things are divided first into ten equal Parts, and these Parts so divided we call Primes, and Secondly, we divide also each of the former Primes into other ten equal Parts, and every one of these Divisions we call Seconds; and Thirdly, we divide each of the said Seconds into ten other equal Parts, and those so divided we call Thirds; and so by decimating the former, and subdecimating these latter, we run on *ad infinitum*.

28. Let a Pound *sterling*, *Troy-weight*, *Averdupois-weight*, *Liquid-measure*, *Dry-measure*, *Long-measure*, *Time*, *Dozen*, or any other Thing or Integer be given to be decimally divided: In this Notion premised, we ought to let the first Division be Primes, the next Division Seconds, the next Thirds, &c. so one Pound *sterling* being 20 Shillings, which divided into ten equal Parts, the Value of each Part will be two Shillings, therefore one Prime of a Pound *sterling* will stand thus (.1) which is in Value 2 Shillings, 3 Primes will stand thus (.3) and that is in Value 6 Shillings. Again, a Prime, or .1 being divided into ten equal Parts, each of those Parts will be one Second, and is thus expressed (.01) and its Value will be found 2d Farthing and  $\frac{1}{8}$  of a Farthing; and so will .05 signify one Shilling or five Second: And if .01 be divided into ten other equal Parts, each of those Parts so divided will be Thirds, and will stand thus .001, and its Value will be found to be .96 of a Farthing, or  $\frac{1}{8}$  of a Farthing, and .009 Thirds will be 2d. and .64 of a Farthing, or  $\frac{1}{8}$  of a Farthing; so that .375d. will be found to represent 7s. 6d. for the 3 Primes are 6s. and the 7 Seconds are 1s. 4d. and  $\frac{1}{8}$  of a Penny, and the 5 Thirds are 1 Penny  $\frac{1}{8}$  of a Penny, both which added together make 7s. 6d.

29. If you put any Bulk or Body representing an Integer, and it be decimally divided, then the Parts in the first

Decimation

Decimation are Primes, the next Seconds, and the next Decimation is Thirds, the next Fourths, &c. As let there be given a Bullet of Lead, or such like, whose Weight let it be 50lb Troy, this is called an Unit, Integer or Thing; then will the like Weight and Matter make 10 other, the which together will be equal to 50lb and will weigh each of them 5lb a-piece; take of the same Matter, and equal to 5lb make 10 more, then each of those weigh 10 Ounce a-piece; also, if again you take 6 Ounces and thereof make 10 other small Bullets, each of them will weigh 12 Pennyweight Troy; and thus have you made Primes, Seconds and Thirds, in respect of the Integer, containing 50lb Troy weight; so that 5 Primes is equal to the half Mass, and 2 Primes, and 5 Seconds is a quarter of the Mass; and therefore one of the first Division, two of the second Division, and five of the third Division, will be equal in Weight to half a quarter of the Mass, and contains 6lb 3oz.

30. When a decimal Fraction followeth a whole Number, you are to separate or part the Decimal from the whole Number by a Point or Prick; so if .75 follow the whole Number 32, set them thus, 32.75. You shall find that divers Authors have diverse Ways in expressing mix'd Numbers as thus, 32|75, or 32| $\frac{75}{100}$ , or 32|75 but you will find that 32.75, thus placed and expressed, is the fittest for Calculation.

31. A mix'd Number hath two Parts, the whole and the broken; the whole is that which is composed of Integers and the broken is a Fraction annexed thereunto. So the mix'd Number  $36\frac{8}{12}$  being given, we say, that 36 is the whole Number, which is composed of Integers, and the  $\frac{8}{12}$  is the broken Number annexed, which sheweth that one of the former Integers (of that 36) being divided into 12 Parts, this  $\frac{8}{12}$  doth express 8 of those 12 Parts more belonging to the said 36 Integers.

32. Denominative Numbers are of one, or of many, and those are of divers Sorts and Kinds, viz. Singular, called Unit, as 1; and Plural a Multitude, as 2, 3, 4, 5: Single, of one Kind only called Digits, as 1, 2, 3, 4, 5, 6, 7, 8, 9, and Compounds of many, 10, 11, 12, &c. 102, 367, &c.

Proportional, as Single, Multiple, Double, Triple, Quadruple &c. Denominate, as Pounds Shillings, Pence; Undernominate, as 1, 2, 3, &c. Perfect, as 6, 28

26, 8128, 130816, 20999128, &c. whose Parts are equal to the Numbers; imperfect, unequal, and more than the Sum, as 12, to 1, 2, 3, 4, 6; Imperfect, unequal, and less than the Sum, as 8 to 1, 2, 4. : Numbers commensurable and incommensurable, as 12 and 9 are commensurable, because 3 measures them both; but 6 and 17 are incommensurable, because no one common Number or Measure can measure them; Linear, in form of a Line, as ..... : superficial, in form of a Superficies or Plane, as :::: or ::::: and Number cubical or solid, in form of a Cube: These two latter are otherwise called figurative Numbers. There are also other Numbers called tabular, as Sines, Tangents, Secants &c. others that be called Logarithmical, or borrowed Numbers, fitted to Proportion for Ease, and speedy Calculation of all manner of Questions.

## CHAP. II.

*Of the natural Divisions of Integers, and the several Denominations of the Parts.*

1. **A**ND that we may advance methodically herein, we will begin with the main Pillars on which Arithmetick is founded, *viz.* the several Species of that Art: But first,

*Of Money, Weights, &c.*

2. The least Denomination or Fraction of Money used in England is a Farthing, from which is produced the following Table, called the Table of Coin, &c.

And therefore,

1 Farthing	} <i>NEW</i>	1 Farthing	} <i>OLD</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
4 Farthings		1 Penny		1	20	12	4
12 Pence		1 Shilling		1	20	240	960
20 Shillings		1 Pound			1	12	48

The first of these Tables, *viz.* that on the left Hand, is plain and easy to be understood, and therefore wants no Direction. In the second Table above the Line, you have 1*l.* 20*s.* 12*d.* 4*qrs.* where by is meant, that 1 Pound is equal to 20 Shillings, and 1 Shilling is equal to 12 Pence, and 1 Penny equal to 4 Farthings; under that Line is 1*l.* 20*s.* 240*d.* 960*qrs.* which signifies 1*l.* to contain 20 Shillings, or 240 Pence, or 960 Farthings; in the second Line below that is 1*s.* 12*d.* 48*qrs.* the first standing under the Denomination

nation of Shillings, whereby is to be noted, that 1 Shilling is equal to 12 Pence or 48 Farthings; and likewise that below. that one Penny is equal in Value to four Farthings. Understand the like Reason in all the following Tables of Weight, Measure, Time, Motion and Dozen. (See the Appendix to *Dilworth's Arithmetick for the Irish Weights and Measures, &c.*

### Of Troy-weight.

3. The least Fraction or Denomination of Weight used in *England*, is a Grain of Wheat gathered out of the Middle of the Ear, and well dried; from whence are produced these following Tables of Weight, called *Troy weight*.

32 Grains of Wheat	} make {	24 Artificial Grains
24 Artificial Grains		1 Penny-weight
20 Penny-weight		1 Ounce
12 Ounces		1 Pound Troy-weight.

And therefore,  $\text{lb}$     $\text{oz.}$     $\text{pwt.}$     $\text{gr.}$

1	12	20	24
<hr/>			
1	12	240	5760
	1	20	480
		1	24

*Troy weight* serveth only to weigh Bread, Gold, Silver and Electuaries; it also regulateth and prescribeth a Form how to keep the Money of *England* at a certain Standard. But Bread in *Ireland* is now weighed by *Averdupois-weight* and the Ounce is divided into eight Drams.

### Of Apothecaries-weight

4. The Apothecaries have their Weights deduced from *Troy-weight*, a Pound *Troy* being the greatest Integer, Table of whose Division and Subdivision followeth, viz.

And therefore,  $\text{lb}$     $\text{oz.}$     $\text{dr.}$     $\text{scr.}$     $\text{gr.}$

1 Pound	} make {	12 Ounces	} 1	12	8	3	20
1 Ounce		8 Drams		1	12	99	288
1 Dram		3 Scruples		1	8	20	480
1 Scruple		20 Grains		1	3	60	20

5. Thus much concerning *Troy-weight* and its derivative Weights; besides which, there is another Kind of Weight used in *England*, known by the Name of *Averdupois-weight*. 1 Pound of which is equal to 14 Ounces 12 Penny-weight *Troy-weight*, and it serves to weigh all Kinds of Groceries, wares, and also Butter, Cheele, Flesh, Wax, Tallow, Rosin, Pitch, Lead, &c. the Table of Weight is as followeth.

A Table



*A Table of Averdupois-weight.*

Quarters of a Dram	1 Dram
6 Drams	1 Ounce
6 Ounces	1 Pound
3 Pounds	1 Quarter of a Hundred
4 Quarters	1 Hundred Wt. or 112 lb
6 Hundred	1 Tun

And therefore,

<i>Tun</i>	<i>C.</i>	<i>qrs.</i>	<i>lb.</i>	<i>oz.</i>	<i>drams</i>	<i>grs.</i>
1	20	4	28	16	16	4
1	20	80	2240	35840	573440	2293760
	1	4	112	1792	28672	114688
		1	28	448	7168	28672
			1	16	256	1024
				1	16	64
					1	4

Wooll is weighed with this Weight, but only the Divisions are not the same.

7 Pounds	1 Clove	} make	1 Clove	* In Ireland, 16 lb make a Stone of Wool, to be marked Wool, on the Bags, not Wool.
2 Cloves	1 Stone		1 Stone	
2 Stones	1 Todd		1 Todd	
6 Todd	1 Wey		1 Wey	
2 Weys	1 Sack		1 Sack	
2 Sacks	1 Last		1 Last	

And therefore,

<i>last</i>	<i>sack</i>	<i>wey</i>	<i>todd</i>	<i>stone</i>	<i>cloves</i>	<i>lb</i>
1	12	2	6½	2	2	7
1	12	24	156	312	624	4368
	1	2	13	26	52	364
		1	6½	13	26	182
			1	2	4	28
				1	2	14
					1	7

*Note.* That in some Counties the Wey is 256 lb *Averdupois*, as is the *Suffolk Wey*; but in *Essex* there is 336 lb a Wey.

6. The least denomitive Part of Liquid Measure is a Pint, which was formerly taken from *Troy-weight*, (1 Pound of Wheat *Troy-weight* making a Pint of Liquid Measure) but since, by a late Act of Parliament, to prevent Fraud in the Excise, the Pint Beer Measure is to contain 35½ solid Inches, and the Pint of Wine 28½ the like Inches, &c.

*A Table*

## A Table of Liquid Measure.

35 $\frac{1}{4}$ Cubical Inches	} make	1 Pint Beer Measure
28 $\frac{7}{8}$ Cubical Inches		1 Pint Wine Measure
2 Pints		1 Quart
2 Quarts		1 Pottle
† 2 Pottles		1 Gallon
8 Gallons		1 Firkin of Ale, Soap or Herrings
9 Gallons		1 Firkin of Beer
10 Gallons and half		1 Firkin of Salmon or Eels
2 Firkins		1 Kilderkin
2 Kilderkins		1 Barrel
42 Gallons		1 Tierce of Wine
63 Gallons		1 Hogshhead
2 Hogsheds		1 Pipe or Butt
2 Pipes or Butts		1 Tun of Wine

† The *Irish* Gallon contains 217  $\frac{5}{16}$  cubic Inches, the Pint 27.2, and 10 Gallons make a Firkin of Ale or Beer, 4 Firkins a Barrel, and 8 Barrels a Tun in *Ireland*.

And therefore,

Tuns pipes bbl. gal. pint

1	2	2	63	
1	2	4	252	201
	1	2	126	100
		1	63	50
			1	

7. The least denominative Part of Dry Measure is all a Pint, and this is likewise taken from *Troy-weight*.

## A Table of Dry Measure.

1 Pound Troy	} make	1 Pint
2 Pints		1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
2 Gallons		1 Peck
4 Pecks		1 Bushel
4 Bushels		1 Comb, or <i>Irish</i> Barrel
2 Combs		1 Quarter
4 Quarters		1 Chaldron
5 Quarters		1 Wey
2 Weys		1 Last

And therefore,

last wey		qrs.	combs	busb.	pecks	gal.	pints
1	2	5	2	4	4	2	8
1	2	10	20	80	320	640	5120
	1	5	10	40	160	320	2560
		1	2	8	32	64	512
			1	4	16	32	256
				1	4	8	64
					1	2	16
						1	8

8. The least denominative Part of Long-measure is a Barly-corn well dried, and taken out of the Middle of the Ear, whose Table of Parts followeth.

3 Barly-corns	}	make	1 Inch
12 Inches			1 Foot
3 Feet			1 Yard
3 Feet 9 Inches			1 Ell <i>Englisch</i>
6 Feet			1 Fathom
5 Yards $\frac{1}{2}$ in <i>England</i>			
But 7 Yards of <i>Irish</i>	}	make	1 Pole, Perch or Rod
Plantation Measure			
40 Poles or Perches			1 Furlong
8 Furlongs	}	make	1 Mile

And therefore,

mile furl.		poles	yards	feet	inches	barly-corns
1	8	40	$5\frac{1}{2}$	3	12	3
1	8	320	1760	5280	63360	190080
	1	40	220	660	7920	23760
		1	$5\frac{1}{2}$	$16\frac{1}{2}$	198	594
			1	3	36	108
				1	12	36
					1	3

And note, that the Yard, as also the Ell, is usually divided into Quarters, and each Quarter into 4 Nails.

Note also, that a geometrical Pace is five Feet, and there are 1056 such Paces in an *Eng.* Mile, 1344 in an *Irish*.

9. The Parts of the superficial Measures of Land are such as are mentioned in the following Table, viz.

A Table of Land-measure.

40 Square Poles or Perches	}	make	1 Rood, or Quarter of an Acre
4 Roods			1 Acre

By the foregoing Table of *Long-measure* you are informed what a Pole or Perch is; and by this, that 40 square Perches is a Rood: Now a square Perch is a Superficies very aptly resembled by a square Trencher, every Side thereof being a Perch in Length, 40 of them is a Rood, and 4 Roods an Acre; so that a Superficies that is 40 Perches long and 4 broad is an Acre of Land, the Acre containing in all 160 square Perches.

ro. The least denominative Part of Time is one Second, the greatest Integer being a Year, from whence is produced this

*Table of Time.*

60 Seconds	} make	1 Minute
60 Minutes		1 Hour
24 Hours		1 Day natural
7 Days		1 Week
4 Weeks		1 Month
13 Months, 1 Day, 6 Hours		1 Year

But the Year is usually divided into twelve unequal Calendar Months, whose Names, and the Number of Days they contain, are as follow, viz.

Days		Days		Days, and 6 Hours;	
January 31	July 31	August 31	September 30	October 31	November 30
February 28	August 31	September 30	October 31	November 30	December 31
March 31	September 30	October 31	November 30	December 31	
April 30	October 31	November 30	December 31		
May 31	November 30	December 31			
June 30	December 31				

So that the Year containeth 365

Days, and 6 Hours; but the 6 Hours are not reckoned, but only every Leap Year, and then there is a Day added to the latter end of February, and then it containeth 29 Days; and that Year is called Leap-year, and containeth 366 Days.

And here note, that as the Hour is divided into 60 Minutes, so each Minute is sub-divided into 60 Seconds, and each Second into 60 Thirds, and each Third into 60 Fourths, &c.

The Tropical Year, by the exactest Observation of the most accurate Astronomers, is found to be 365 Days, 5 Hours, 49 Minutes, 4 Seconds and 21 Thirds.

### C H A P. III.

#### *Of the Species or Kinds of Arithmetick.*

There are several Species of this Art, and which may be termed either Natural, Artificial, Analytical Algebraical, Lineal, or Instrumental; but what we are now to treat upon relates to the single Parts of Natural Arithmetick, so far as concerns Numeration; of which there are also four Kinds, viz. *Addition, Subtraction, Multiplication and Division.*

## C H A P. IV.

*Addition of Whole Numbers.*

**A**DDITION is the Reduction of two or more Numbers, of like Kind, together into one Sum or Total: Or, it is that by which divers Numbers are added together, to the end that the Sum or total Value of them all may be discovered.

The first Number in every *Addition* is called the *Addible Number*; the other, the *Number* or *Numbers* added; and the *Number* invented by the *Addition* is called the *Aggregate* or *Sum*, containing the Value of the *Addition*.

The Collation of the Numbers, is the right placing the Numbers given respectively to each Denomination, and the Operation is the artificial adding of the Numbers given together, in order to the finding out of the Aggregate or Sum.

2. In *Addition* place the Numbers given respectively the one above the other, in such sort, that the like Degree, Place, or Denomination, may stand in the same Series, *viz.* Units under Units, Tens under Tens, Hundreds under Hundreds, &c. Pounds under Pounds, Shillings under Shillings, Pence under Pence, &c. Yards under Yards, Feet under Feet, &c.

3. Having thus placed the Numbers given (as before) and drawn a Line under them, add them together, beginning with the lesser Denomination, *viz.* at the right Hand; and so on, subscribing the Sum under the Line respectively: As for Example.

Let there be given 3352, and 213, and 133. to be added together. I set the Units in each particular Number under each other, and so likewise the Tens under the Tens, &c. and draw a Line under them, as in the Margent;

then I begin at the Place of Units and add them together upwards, saying, 3 and 3 are 6, and 2 makes 8, which I set under the Line, and 1 under the same Figures added together; then I proceed to the next Place being the Place of Tens, and add them in the same Manner as I did in the Place of Units, saying, 3 and 1 are 4 and 5 are 9, which likewise set under the Line respectively; then I go on to the Place of Hundreds, and add them up as I did the other, saying, 1 and 2 are 3 and 3 are 6, which is also set under the Line; and lastly, I go to the Place of Thousands, and because there are no other Figures to add to the 3, I set it under the Line

Line



Line in its respective Place, and so the Work is finished and I find the Sum of the three given Numbers to be 3698

4 But if the Sum of the Figures of any Series exceedeth ten, or any Number of Tens, subscribe under the same the Excess above the Tens, and for every ten carry one, to be added to the next Series towards the left Hand, and so go on till you have finished your Addition, always remembering, that how great soever the Sum of the Figures of the last Series is, it must all be set down under the Line respectively; so 3678 being given to be added to 2357, set them down as is before directed, and as you see in the Margent, with a Line drawn under them, then I begin and add them together, saying, 7 and 8 are 15, which is 5 above 10, wherefore I set 5 under the Line, and carry 1 for the 10 to be added to the next Series, saying, 1 that I carried and 5 is 6 and 7 are 13, wherefore I set down 3, and carry 1 (for the Ten) to the next Series; then I say, 1 that I carried and 3 are 4 and 6 are 10, now, because it comes to just 10 and no more, I set 0 under the Line, and carry 1 for the 10 to the next and say, 1 that I carried and 2 are 3 and 3 are 6, which I set down in its respective Place; thus the Addition is ended and the total Sum of these Numbers is found to be 6035. Several Examples of this Kind follow.

$$\begin{array}{r}
 \text{Numbers to} \\
 \text{be added} \left\{ \begin{array}{l} 354867 \\ 573846 \\ 785946 \\ 347205 \end{array} \right. \\
 \hline
 \text{Sum } 2061864
 \end{array}$$

$$\begin{array}{r}
 \text{Numbers to} \\
 \text{be added} \left\{ \begin{array}{l} 748647 \\ 465834 \\ 76483 \\ 648400 \end{array} \right. \\
 \hline
 \text{Sum } 1939364
 \end{array}$$

$$\begin{array}{r}
 \text{Numbers to} \\
 \text{be added} \left\{ \begin{array}{l} 45346 \\ 38074 \\ 8437 \\ 923 \\ 76 \end{array} \right. \\
 \hline
 \text{Sum } 92856
 \end{array}$$

5. If the Numbers given to be added are contained under divers Denominations, as of *Pounds, Shillings, Pence* and *Farthings*, or of *Tuns, Hundreds, Quarters, Pounds* &c. then in this Case, having disposed of the Numbers of each Denomination under the others of the like Kind, beginning at the least Denomination (minding how many of one Denomination do make an Integer of the next) and having added them up, for every Integer of the next greater Denomination that you find therein contained, be

an Unit in mind to be added to the said next greater Denomination, expressing the Excess respectively under the Line ; proceed in this manner until your Addition be finished ; the following Example will make the Rule plain to the Learner. Thus these following Sums being given to be added, viz. 136*l.* 13*s.* 04*d.* 2*qrs.* and 79*l.* 07*s.* 10*d.* 3*qrs.* and 33*l.* 18*s.* 09*d.* 1*qr.* alio 15*l.* 09*s.* 05*d.* 0*qrs.* The Numbers being disposed according to Order, will stand as in the Margent ; then I begin at the Denomination of Farthings, and add them up, saying, 1 and 3 are 4 and 2 make 6. Now I consider that 6 Farthings are 1 Penny, 2 Farthings ; wherefore

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
I set down the 2 Farthings in its Place under the Line, and keep 1	136	13	04	2
in mind to be added to the next Denomination of Pence ; then I go on, saying, 1 that I carried and 5 are 6	79	07	10	3
and 9 are 15 and 10 are 25 and 4 are 29 ; now I consider that 29 Pence are 2 Shillings and 5 Pence, therefore I set down 5 Pence in Order under the Line, and keep 2 in mind for 2 Shillings to be added to the Shillings :	33	18	09	1
then I go on saying, 2 that I carried and 9 are 11 and 18 are 29 and 7 are 36 and 13 are 49 ; then I consider that 49 Shillings are 2 Pounds and 9 Shillings, wherefore I set the 9 Shillings under the Line, and carry 2 for the 2 Pounds to the next and last Denomination of Pounds, and proceed, saying, 2 that I carried and 5 make 7 and 3 are 10 and 9 are 19 and 6 are 25 ; then I set down 5 and carry 2 for Tens and proceed, saying, 2 that I carry and 1 is 3 and 3 are 6 and 7 are 13 and 3 make 16, and I set down 6 and carry 1 for the 10, and go on, saying, 1 that I carried and 1 are 2, which I set in its Place under the Line, and the Work is finished ; and thus I find the Sum of the aforesaid Numbers to be 265 <i>l.</i> 9 <i>s.</i> 5 <i>d.</i> 2 <i>qrs.</i>	15	09	05	0
Here is another Example, in the Operation of which the Learner must have an Eye to the Table of <i>Tro-weight</i> ; the Numbers given are 38 <i>lb.</i> 7 <i>oz.</i> 13 <i>pwt.</i> 18 <i>gr.</i> and 50 <i>lb.</i> 10 <i>oz.</i> 10 <i>pwt.</i> 12 <i>gr.</i> and 42 <i>lb.</i> 8 <i>oz.</i> 5 <i>pwt.</i> 16 <i>gr.</i>	265	09	05	2

and in order to the Addition thereof I place them as you see, and proceed to the Operation, saying, 16 and 12

are 28 and 18 are 46 : now because 24 Grains make 1 Penny-weight, 46 Grains are 1 Penny-weight and 22 Grains, therefore I set down 22. and carry 1 for the Penny-weight, and 5 make 6 and 10 are 16 and 13 are 29, which is 1 Ounce and 9 Penny weight ; I set down 9 in its Place under the Line, and carry 1 to the Ounces, saying. 1 that I carry and 8 are 9 and 10 are 19, and 7 are 26, and because 26 Ounces make 2 Pounds 2 Ounces, I set down 2 for the Ounces, and carry 2 to the Pounds, going on, 2 that I carry and 2 are 4 and 8 make 12, that is 2 and go 1 ; then 1 I carry and 4 are 5 and 5 are 10 and 3 are 13, which I set down as in the Margent, and the Work is finished ; and I find the Sum of the said Numbers to amount to 132lb 02oz. 09pwt. 22gr. The Way of proving these, or any Sum in this Rule, is shewed immediately after the ensuing Example.

*Addition of English Money.*

l.	s.	d.	q.	l.	s.	d.	qrs.
436	13	07	1	48	15	11	1
184	09	10	3	76	10	07	3
768	17	04	2	18	00	05	3
584	11	11	0	24	19	09	2
1974	12	09	2	168	06	10	1

*Addition of Troy-weight.*

lb	oz.	pwt.	gr.	lb	oz.	pwt.	gr.
15	07	13	12	145	09	12	18
18	06	04	20	726	08	14	10
11	10	16	18	389	07	06	13
09	04	10	22	83	10	16	20
19	11	18	04	130	00	10	12
22	00	00	05	74	07	15	00
97	05	04	09	1550	08	16	01

*Addition of Apothecaries Weight.*

lb	oz.	dr.	sc.	gr.	lb	oz.	dr.	sc.	gr.
48	07	1	0	14	60	03	4	0	10
74	05	5	2	10	48	10	6	0	14
64	10	7	1	16	34	08	2	1	15
17	08	1	0	11	18	11	2	2	11
34	09	6	1	09	160	07	1	2	15
240	05	6	1	00	35	02	5	1	07
					358	07	7	0	12

*Addition of Averdupois Weight.*

Tuns	C.	qrs.	lb	lb	oun.	dr.
75	13	1	15	36	10	12
48	07	3	21	22	01	13
60	11	1	17	11	07	04
21	07	0	25	15	08	10
12	16	0	11	20	00	09
218	16	0	05	105	13	00

*Addition of Liquid Measure.*

Tuns	pipe	bhd.	gal.	Tuns	bhd.	gal.	pts.
45	1	1	48	30	3	40	4
15	0	1	17	12	2	28	6
38	0	0	47	47	3	60	5
12	1	0	56	57	3	22	3
21	1	1	18	17	0	00	0
133	1	1	60	166	1	26	2

*Addition of Dry Measure.*

Cbal.	qrs.	bufh.	pec.	qrs.	bufh.	pec.	gal.
48	3	7	3	17	3	1	1
13	1	4	0	50	1	3	0
54	0	6	2	14	5	3	1
16	3	6	1	40	2	0	1
40	1	0	1	30	0	3	0
173	3	0	3	152	5	3	1

*Addition of Long Measure.*

Yds.	qrs.	nails	Ells	qrs.	nails
35	3	3	56	1	3
14	1	2	13	3	2
74	2	3	48	2	1
38	0	1	50	0	2
30	1	0	74	2	0
15	0	0	17	1	0
208	1	1	260	1	0

*Addition*

## Addition of Land Measure.

Acre	Rood	Perch	Acre	Rood	Perch
12	3	18	86	1	36
14	0	24	47	3	24
30	2	19	73	2	28
48	3	30	60	0	07
28	1	38	04	2	08
50	3	26	14	1	14
185	3	35	286	3	37

## The Proof of Addition.

6. *Addition* is proved after this Manner: When you have found out the Sum of the Numbers given, then separate the uppermost Line from the rest, with a Stroke or Dash of the Pen, and then add them all up again as you did before, leaving out the uppermost Line; and having it done, add the new invented Sum to the uppermost Line you separated, and if the Sum of these two Lines be equal to the Sum first found out, then the Work is performed true; otherwise not. As for Example: Let us prove the first Example of *Addition of Money*, whose Sum we found to be 265 *l.* 9 *s.* 5 *d.* 2 *qrs.* and which we prove thus: Having separated

the uppermost Number from the rest by a Line, as you see in the Margent, then I add the same together again, leaving out the said uppermost Line, and the Sum thereof I set under the first Sum or true Sum, which doth amount to 128 *l.* 16 *s.* 1 *d.* 0 *qrs.*, then again I add the new Sum to the uppermost Line, that before was separated from the rest, and the Sum of these two is 265 *l.* 9 *s.* 5 *d.* 2 *qrs.* the same with the first Sum, and therefore I conclude that the Operation was rightly performed.

7. The main End of *Addition*, in Questions resolvable thereby, is to know the Sum of several Debts, Parcels, Measures, &c. Some Questions may be these that follow.

*Quest. 1.* There was an old Man whose Age was required; to which he replied, I have seven Sons, each living two Years between their Birth, and in the 44th Year of my Age my eldest Son was born, which is now the Age of the youngest. I demand what was the old Man's Age.

Now to resolve this Question, first set down the Father's Age at the Birth of his first Child, which was 44; then



the Difference between the oldest and the youngest, 44  
 which is 12 Years, and then the Age of the youngest, 12  
 which is 44; and then add them all together, and 44  
 their Sum is 100, the compleat Age of their Father. 100

*Quest. 2.* A Man lent his Friend, at several Times, these  
 several Sums, viz. at one Time 63<sup>l</sup>. at another Time 50<sup>l</sup>  
 at another Time 48<sup>l</sup>. at another Time 156<sup>l</sup>. Now I desire  
 to know how much was lent him in all?

Set the Sums lent under one another, as you see in 63  
 the Margent, and then add them together, and you 50  
 will find their Sum to amount to 317<sup>l</sup>. which is the 48  
 Total of all the several Sums lent, and so much is 156  
 due to the Creditor. 317

*Quest. 3.* There are two Numbers, the least whereof is  
 40, and their Difference 14. I desire to know  
 what is the greater Number, and also what is 40  
 the Sum of them both? First set down the least, 14  
 viz. 40, and 14 the Difference, and add them  
 together, and their Sum is 54 for the greatest 54  
 Number; then I set 40 (the least) under 54 (the 40  
 greatest) and add them together, and their Sum  
 is 94, equal to the greatest and least Numbers. Sum 94

## C H A P. V.

## Of Subtraction of Whole Numbers.

**SUBTRACTION**, is the taking a lesser Number out of  
 a greater of a like Kind, whereby to find out a third  
 Number, being or declaring the Inequality, Excess, or  
 Difference between the Numbers given; or, *Subtraction* is  
 that by which one Number is taken out of another Num-  
 ber given, to the End that the Residue or Remainder may  
 be known, which Remainder is also called the Rest, Re-  
 mainder, or Difference of the Numbers given.

2. The Number out of which *Subtraction* is to be made  
 must be greater, or at least equal with the other Number  
 given; the higher Number is called the *Major*, and the  
 lower, *Minor*; and the Operation of *Subtraction* being finish-  
 ed, the Rest or Remainder is called the *Difference* of the  
 Number given.

3. In *Subtraction*, place the Numbers given respectively  
 the one under the other, in such Sort as like Degrees,  
 Places, or Denominations may stand in the same Series,  
 viz. Units under Units, Tens under Tens, Pounds under  
 Pounds,

Pounds, &c. Feet under Feet, and Parts under Parts, &c. This being done, draw a Line underneath, as in *Addition*.

4. Having placed the Numbers given as is before directed, and drawn a Line under them, subtract the lower Number (which in this Case must always be less than the uppermost) out of the higher Number, and subscribe the Difference or Remainder respectively below the Line, and when the Work is finished, the Number below the Line will give you the Remainder.

As for Example: Let 364521 be given to be subtracted from 795836, I set the lesser under the greater, as in the Margent, and draw a Line under them; then beginning at the Right Hand, I say, 1 out of 6 364521 and there remains 5, which I set in order under the Line; then I proceed to the next, saying, 2 431315 from 3 rests 1, which I note also under the Line; and thus I go on till I have finished the Work, and then I find the Remainder or Difference to be 431315.

5. But if it so happen (as commonly it doth) that the lowermost Number or Figure is greater than the uppermost: then in this Case add ten to the uppermost Number, and subtract the said lowermost Number from their Sum, and the Remainder place under the Line; and when you go to the next Figure below pay an Unit, by adding it thereto, for the ten you borrowed before, and subtract that from the higher Number of Figures, and thus go on till your Subtraction be finished. As for Example: Let 437503 be given, from whence it is required to subtract 153827,

dispose of the Numbers as is before directed, and as you see in the Margent; then I begin, saying, 7 from 3 I cannot, but (adding 10 thereto) I say 153827 7 from 13 and there remains 6, which I set under the Line in Order; then I proceed to the next Figure, saying, 1 that I borrowed and 2 is 3 from

0 I cannot, but 3 from 10 and there remains 7, which I likewise set down as before; then 1 that I borrowed and 8 is 9 from 5 I cannot, but 9 from 15 and there remains 6; then 1 I borrowed and 3 is 4 from 7 and there remains 3; then 5 from 3 I cannot, but 5 from 13 and there remains 8; then 1 I borrowed and 1 are 2 from 4 and there rests 2, and thus the Work is finish'd: After these Numbers are subtracted one from another, the Inequality, Remainder, Excess or difference is found to be 283676. Examples for thy farther Experience may be these that follow.

From 3469916  
Take 738642  
Rest 2731274

From 361577  
Take 5864  
Rest 355713

6. If the Sum or Number to be subtracted is of several Denominations, place the lesser Sum below the greater, and in the same Rank and Order as is shewed in *Addition* of the same Numbers; then begin at the right Hand, and take the lower Number out of the uppermost, if it be lesser; but if it be greater, than the uppermost, then borrow an Unit from the next greater Denomination, and turn it into the Parts of the lesser Denomination, and add those Parts to the uppermost, noting the Remainder below the Line; then proceed and pay one to the next Denomination for that which you borrowed before; and proceed in this Order till the Work is finished. An Example of this Rule followeth: Let 375*l.* 13*s.* 7*d.* 1*qr.* be given, from whence let it be required to subtract 57*l.* 16*s.* 3*d.*

3*qrs.* In order whereunto I place the *l. s. d. qrs.*  
 Numbers as you see in the Margent; and 375 13 07 1  
 thus I begin at the least Denomination, 57 16 03 2  
 saying, 2 from 1 I cannot, therefore I borrow one Penny from the next Denomination, and turn it into Farthings, which is 4, and adding 4 to 1, which is 5, I say, but 2 from 5 and there remains 3, which I put under the Line; then going on I say, 1 that I borrowed and 3 is 4 from 7 and there rests 3; then going on I say, 16 from 13 I cannot, but borrowing 1 Pound, and turning it into 20 Shillings, I add it to 13, and that is 33 wherefore I say, 16 from 33 and there remains 17, which I set under the Line, and go on, saying, 1 that I borrowed and 7 is 8 from 5 I cannot; but 8 from 15 and there remains 7, and the 1 that I borrowed and 5 is 6 from 7 there rests 1, and 0 from 3 rests 3, and the Work is done: And I find the Remainder or Difference to be 317*l.* 17*s.* 3*d.* 3*qrs.*

Another Example of *Troy-weight* may be this, I would subtract 17*lb* 10*oz.* 11*pwt.* 20*gr.* from 24*lb* 5*oz.* 00*pwt.* 08*gr.* I place the Numbers according to the Rule, and begin saying, 20 *lb oz. pwt. gr.*  
 from 8 I cannot, but I borrow 1 24 05 00 08  
 Penny-weight, which is 24 Grains and 17 10 11 20  
 add them to 8, and these are 32, 06 06 08 12  
 wherefore I say 20 from 32 rests 12; then 1 that I borrowed and 11 is 12 from 00 I cannot, but 12 from 20 (borrowing 1 Ounce, which is 20 Penny weight) and there remains 8;

then 1 that I borrowed and 10 is 11 from 5 I cannot, but 11 from 17 and there refts 6; then 1 that I borrowed and 7 is 8 from 4 I cannot, but 8 from 14 and there refts 6; then 1 that I borrowed and 1 is 2 from 2 and there refts nothing; fo that I find the Remainder or Difference to be 6<sup>lb</sup> 6oz. 8<sup>pwt.</sup> 12<sup>gr.</sup>

7. It many times happeneth that you have many Sums or Numbers to be subtracted from one Number; as, suppose a Man should lend his Friend a certain Sum of Money, and his Friend hath paid him part of his Debt at several Times, then before you can conveniently know what is still owing, you are to add the several Number or Sums of Payment together, and subtract their Sum from the whole Debt, and the Remainder is the Sum due to the Creditor: As suppose *A* lendeth to *B* 564<sup>l.</sup> 16<sup>s.</sup> 10<sup>d.</sup> and *B* hath repaid

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Lent	564	16	10
Paid at several Payments.	79	16	08
	163	18	11
	241	15	08
Paid in all	485	11	03
Remains	79	05	07

him 79<sup>l.</sup> 16<sup>s.</sup> 8<sup>d.</sup> at one Time, and 163<sup>l.</sup> 18<sup>s.</sup> 11<sup>d.</sup> at another Time, and 241<sup>l.</sup> 15<sup>s.</sup> 8<sup>d.</sup> at another Time; and you would know how the Account standeth between them, or what is more due to *A*. In order whereunto I first set down the Sum which *A* lent, and draw a Line

underneath it, then under that Line I set the several Sums of Payment, as you see in the Margent; and having brought the several Sums of Payment into one Total, by the 5th Rule of the 4th Chapter foregoing, I find their Sum amounteth to 485<sup>l.</sup> 11<sup>s.</sup> 3<sup>d.</sup> which I subtract from the Sum first lent by *A*, by the 6th Rule of this Chapter, and I find the Remainder to be 79<sup>l.</sup> 5<sup>s.</sup> 7<sup>d.</sup> and so much is still due to *A*.

When the Learner hath good Knowledge of what hath been already delivered in this and the foregoing Chapters, he will with Ease understand the Manner of working the following Examples.

*Subtraction of Whole Numbers.*

	<i>l.</i>	<i>s.</i>	<i>d.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
Borrowed	374	10	03		700	10	11	2
Paid	79	15	11		9	03	11	3
Remains	294	14	04		691	06	11	3
	<i>l.</i>	<i>s.</i>	<i>d.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
Borrowed	1000	00	00		711	03	00	0
Paid	19	00	06		11	13	00	1
Remains	980	19	06		699	09	11	3

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
Borrowed	3500	00	00	0
Paid at several Payments	170	10	00	0
	361	13	10	1
	590	03	04	3
	73	04	11	3
Paid in all	1195	12	02	3
Remains due	2304	07	09	1

*Subtraction of Troy-weight.*

	<i>lb</i>	<i>oz.</i>	<i>pwt.</i>	<i>gr.</i>
Bought	174	00	13	00
Sold	78	04	16	15
Remains	95	07	16	09
Bought	470	10	13	00
	60	00	00	00
Sold at several Times	35	10	18	00
	16	07	09	08
	48	04	00	00
	61	11	19	23
	23	00	00	00
Sold in all	245	10	07	07
Remain unfold	225	00	05	17

*Subtraction of Apothecaries Weight.*

	<i>lb</i>	<i>oz.</i>	<i>dr.</i>	<i>sc.</i>	<i>gr.</i>	<i>lb</i>	<i>oz.</i>	<i>dr.</i>	<i>sc.</i>	<i>gr.</i>
Bought	12	04	3	0	00	20	00	1	0	07
Sold	8	05	1	1	15	10	00	1	2	12
Rem.	3	11	1	1	05	9	11	7	0	15

*Subtraction of Averdupois-weight*

	<i>C. grs.</i>	<i>lb</i>	<i>Tun</i>	<i>C. grs.</i>	<i>lb</i>	<i>oz.</i>	<i>dr.</i>
Bought	35	0	15	5	07	1	10
Sold	16	2	20	3	17	1	16
Remain	18	1	23	1	09	3	22

*Subtraction of Liquid Measure.*

	<i>tu.</i>	<i>bbd.</i>	<i>gal.</i>	<i>tu.</i>	<i>bbd.</i>	<i>gul.</i>	<i>pints</i>
Bought	40	1	30	60	3	42	4
Sold	16	1	40	15	3	46	6
Remain	23	3	53	44	3	58	6



## Subtraction of Dry Measure.

	chal.	qrs.	bussh.	pecks		cha.	qrs.	bussh.	pecks
Bought	100	0	0	0		73	2	3	2
Sold	54	1	4	3		46	2	3	3
Remain	45	2	3	1		26	3	7	3

## Subtraction of Long Measure.

	yds.	qrs.	nails		yds.	qrs.	nails
Bought	160	0	0		344	0	1
Sold	64	1	2		177	1	2
Remain	95	2	2		166	2	2

## Subtraction of Land Measure.

	acres	rood	perch.		acres	rood	perches
Bought	140	2	13		600	0	00
Sold	70	3	12		54	0	16
Remain	69	3	01		545	3	24

## The Proof of Subtraction.

8. When your Subtraction is ended, if you desire to prove the Work, whether it be true or not, then add the Remainder to the *minor* Number, and if the Aggregate of these two be equal to the *major* Number, then is your Operation true, otherwise false: Thus let us prove the first Example of the fifth Rule of this Chapter, where, after Subtraction is ended, the Numbers stand as in the Margent, the Remainder or Difference being 283676: Now to prove the Work, I add the said Remainder 283676 to the *minor* Number 153827, by the fourth Rule of the foregoing Chapter, and I find the Sum or Aggregate to be 437503, equal to the *major* Number, or Number from whence the lesser is subtracted. See the Work in the Margent.

The Proof of another Example, may be of the first Example of the 6th Rule of this Chapter, where it is required, to subtract 57*l*. 16*s*. 3*d*. 2*qrs*. from 375*l*. 13*s*. 7*d*. 1*qr*. and by the Rule I find the Remainder to be 317*l*. 17*s*. 3*d*. 3*qrs*.

Now to prove it, I add the said Remainder 317*l*. 17*s*. 3*d*. 3*qrs*. to the *minor* Number 57*l*. 16*s*. 3*d*. 2*qrs*. and their Sum is 375*l*. 13*s*. 7*d*. 1*qr*. equal to the *major* Number which proves the Work to be true; but if it had happened to be either more or less than the said *major* Number, then the Operation hath been false.

9. The

9. The general Effect of Subtraction, is, to find the Difference or Excess between two Numbers, and the Rest when a Payment is made in part of a greater Sum, the Date of Books printed, the Age of any Thing, by knowing the present Year, and the Year wherein they were made, created, or built, and such like.

The Questions appropriated to this Rule are such as follow.

*Quest. 1.* What difference is there between one Thing of 125 Foot long, and another of 66 Foot long?

To resolve this Question, I first set down the *major* or greater Number 125, and under it the *minor* or lesser Number 66, as is directed in the third Rule of this Chapter, and according to the fourth Rule of the same, I subtract the *minor* from the *major*, and the Remainder, Excess, or Difference I find to be 59. See the Work in the Margent:

*Quest. 2.* A Gentleman having owed a Merchant 365*l.* whereof he hath paid 278*l.* What more doth he owe?

To give an Answer to this Question, I first set down the *major* Number 365*l.* and under it I place 278 the *minor*, and subtract the one from the other, whereby I discover the Excess, Difference or Remainder to be 87; and so much is still due to the Creditor, as per the Margent.

*Quest. 3.* An Obligation was written, a Book printed, a Child born, a Church built, or any other Thing made in the Year of our Lord 1572, and now we account the Year of our Lord 1756, the Question is, to know the Age of the said Things, that is, how many Years are passed since the said Things were made? I say, if you subtract the lesser Number 1572, from the greater 1756, the Remainder will be 184, and so many Years are passed since the making of the said Things; as by this Work in the Margent.

*Quest. 4.* There are three Towns lying in a straight Line, viz. London, Huntingdon and York, now the Distance between the farthest of these Towns, viz. London and York, is 151 Miles, and from London to Huntingdon is 49 Miles, I demand how far is it from Huntingdon to York?

To resolve this Question, subtract 49 the Distance between London and Huntingdon, from 151, the Distance between London and York, and the Remainder is 102, for the true Distance between Huntingdon and York. See the Work in the Margent.

## C H A P. VI.

*Multiplication of Whole Numbers.*

1. **M**ULTIPLICATION is performed by two Numbers of like Kind, for the Production of a third, which shall have Reason to the one as the other hath to the Unit, and in Effect is a most brief and artificial *Compound Addition* of many equal Numbers of like Kind into one Sum. Or, *Multiplication* is that by which we multiply two or more Numbers, the one into the other, to the end that their Product may come forth, or be discovered.

Or, *Multiplication* is the increasing of any one Number by any other, so often as there are Units in that Number, by which the other is increased; or by having two Numbers given, to find a third which shall contain one of the Numbers as many Times as there are Units in the other.

2. *Multiplication* hath three Parts. First, the Multiplicand, or Numbers to be multiplied. Secondly, the Multiplier, or Number given, by which the Multiplicand is to be multiplied. And Thirdly, the Product, or Number produced by the other two, the one being multiplied by the other; as if 8 were given to be multiplied by 4, I say 4 times 8 is 32; here 8 is the Multiplicand, and 4 is the Multiplier, and 32 is the Product.

3. *Multiplication* is either *Single*, by one Figure; or *Compound*, that consists of many.

*Single Multiplication* is said to consist of one Figure, because the Multiplicand and Multiplier consist each of 'em of a Digit, and no more; so that the greatest Product that can arise by *Single Multiplication* is 81, being the Square of 9; and *Compound Multiplication* is said to consist of many Figures, because the Multiplicand or Multiplier consists of more Places than one; as if I were to multiply 436 by 6: It is called *Compound*, because the Multiplicand 436 is of more Places than one, viz. 3 Places.

4. The Learner ought to have all the Varieties of *Single Multiplication* by Heart, before he can well proceed any farther into this Art, it being of most excellent Use, and none of the following Rules in *Arithmetick* but what have a principal Dependence thereupon.

MUL

## MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The Use of the precedent Table is this : In the uppermost Line or Column you have expressed all the Digits from 1 to 9, and likewise beginning at 1 and going downwards in the Side Column, you have the same ; so that if you would know the Product of any two single Numbers multiplied by one another, look for one of them (which you please) in the uppermost Column, and for the other in the side Column, and running your Eye from each Figure along their respective Columns in the common Angle (or Place) where these two Columns meet, there is the Product required. As for Example, I would know how much is 8 times 7 ; first, I look for 8 in the uppermost Column, and 7 in the side Column ; then do I cast my Eye from 8 along the Column downwards from the same, and likewise from 7 in the side Column, I cast my Eye from thence toward the right Hand, and find it to meet with the first Column at 56, so that I conclude 56 to be the Product required, &c.

5. In *Compound Multiplication*, if the Multiplicand consists of many Places, and the Multiplier of but one Figure, first set down the Multiplicand, and under it place the Multiplier in the place of Units, and draw a Line underneath them ; begin then and multiply the Multiplier into every particular Figure of the Multiplicand, beginning at the Place of Units, and so proceed towards the left Hand, setting each particular Product under the Line, in order as you proceed ; but if any of the Products exceed 10, or any Number of Tens, set down the Excess, and for every 10 carry an Unit to be added to the next Product, always remembering to set down the total Product of the last Figure ; which Work being finished, the Sum or Number placed under the Line shall be the true and total Product required.



As for Example, I would multiply 478 by 6; first set down 478, and underneath it 6, in the place of Units, and draw a Line underneath them, as in the Margin, then begin, saying 6 times 8 is 48, which is 8 above four Tens, therefore I set down 8 (the Excess) and beat 4 in mind for the 4 Tens; then I proceed, saying, 6 times 7 is 42, and 4 that I carried is 46, I then set down 6 and carry 4, and go on, saying, 6 times 4 is 24, and 4 that I carried is 28, and because it is the last Figure I set it all down, and so the Work is finished, and the Product is found to be 2868, as was required.

6. When in *Compound Multiplication* the Multiplier consisteth of divers Places, then begin with the Figure in the Place of Units in the Multiplier, and multiply it into all the Figures in the Multiplicand, placing the Product below the Line, as was directed in the last Example; then begin with the Figure of the second Place of the Multiplier, *viz.* the Place of Tens, and multiply it likewise into the whole Multiplicand (as you did the first Figure) placing its Product under the Product of the first Figure; do in the same manner by the third, fourth and fifth, &c. until you have multiplied all the Figures of the Multiplier particularly into the whole Multiplicand, still placing the Product of each particular Figure under the Product of its precedent Figure; herein observing the following Caution.

*A Caution.* In the placing of the Product of each particular Figure of the Multiplier, you are not to follow the 2d Rule of the 4th Chapter, *viz.* to place Units under Units, and Tens under Tens, &c. but to place the Figure or Cypher in the place of Units of the second Line under the second Figure or Place of Tens in the Line above it, and the Figure or Cypher in the Place of Units in the third Line under the place of Tens in the second Line, &c. observing this Order till you have finished the Work, still placing the first Figure of every Line or Product under the second Figure or Place of Tens in that which was above it, and having so done, draw a Line under all these particular Products and add them together; so shall the Sum of all these Products be the total Product required.

As if it were required to multiply 764 by 27, I set 'em down the one under the other, with a Line drawn underneath them, then I begin, saying, 7 times 4 is 28, then I set down 8 and carry 2; then I say, 7 times 6 is 42, and 2 that I carried is 44, that is 4 and go 4; then 7 times 7 is 49, and 4 that I carry is 53, which I set down because I have not another



Another Figure to multiply ; thus have I done with the 7 ; then I begin with the 2, saying, 2 times 4 is 8, which I set down under (4) the second Figure or place of Tens in the Line above it, as you may see in the Margent ; then I proceed, saying, 2 times 6 is 12, that is 2 and carry 1, then 2 times 7 is 14, and 1 that I carry is 15, which I set down, because it is the Product of the last Figure, so that the Product of 764 by 7 is 5348, and by 2 is 1528, which being placed the one under the other, as before directed, as you see in the Margent, and a Line drawn under them, and they added together respectively make 20628, the true Product required, being equal to 27 times 764.

Another Example may be this : Let it be required to multiply 5486 by 465, I dispose of the Multiplicand and Multiplier according to the Rule, and begin multiplying the first Figure of the Multiplier, which is (5) into the whole Multiplicand, and find the Product is 27430 ; then I proceed, and multiply the second Figure (6) of the Multiplier into the Multiplicand, and find the Product to amount to 32916, which is subscribed under the other Product respectively ; then do I multiply the third and last Figure (4) of the Multiplier into the Multiplicand, and the Product is 21944, which is likewise placed under the second Line respectively ; then I draw a Line under the said Product, being placed the one under the other (according to Rule) and add them together, and the Sum is 2550990, the true Product sought, being equal to 5486 times 465, or 465 times 5486.

*More Examples in this Rule are these following.*

$$\begin{array}{r}
 430865 \\
 \underline{4739} \\
 3877785 \\
 1292595 \\
 3016055 \\
 1723460 \\
 \hline
 2041869235
 \end{array}$$

$$\begin{array}{r}
 6400758 \\
 \underline{37496} \\
 38404548 \\
 57606822 \\
 25603032 \\
 44805306 \\
 19202274 \\
 \hline
 240002821968
 \end{array}$$

*Compendium in Multiplication.*

7. Altho' the former Rules are sufficient for all Cases in Multiplication, yet because in the Work of Multiplication many times great Labour may be saved, I shall acquaint the

the Learner with some Compendiums in order thereto viz. if the Multiplicand or Multiplier, or both of them

*Si numeris propositis unus vel uterque adjunctos habeat ad dextram circulos, omissis circulis fiat ipsorum numerorum multiplicatio, & facta demum tot insuper integrorum loci accenseantur quot sunt omissi circuli in utraque factore. Clavis Mat. c. 4. 3.*

end with Cyphers, then in your multiplying you may neglect the Cyphers, and multiply only the significant Figures, and to the Product of these significant Figures add so many Cyphers as the Numbers given to be multiplied did end with; that is annex them on the right Hand of the said Product

so shall that give you the true Product required. As if I were to multiply 32000 by 4300, I set them down in order to be multiplied, as you see in the Margent, but neglecting the Cyphers in both Numbers, I only multiply 32 by 43, and the Product I find to be 1376, to which I annex the 5 Cyphers in the Multiplicand and Multiplier, and then it makes 137600000 for the true Product of 32000 by 4300.

8. If in the Multiplier, Cyphers are placed between significant Figures, then multiply only by the significant Figures neglecting the Cyphers; but here special Notice is to be taken of the true placing of the first Figure after the Neglect of such Cypher or Cyphers, and therefore you must observe in what Place of the Multiplier the Figure you multiply by standeth, and set the first Figure of that Product under the same Place of the

*Si intermedio multiplicantis loco circulus fuerit, ille negligitur. Alsted. c. 6. de Arith.*

371568  
40007  
2600976  
1486272  
14865320976

Product of the first Figure of your Multiplier: As for Example, let it be required to multiply 371568 by 40007; first I multiply the Multiplicand by 7, and the Product is 2600976; then, neglecting the Cyphers, I multiply by 4, and that Product is 1486272; now I consider that 4 is the fifth Figure in the Multiplier, therefore I place 2 (the first Figure of the Product by 4) under the fifth place of the first Product by 7, and the rest in Order, and having added them together, the total Product is found to be 14865320976. Other Examples in this Rule are these following.

327586

6030

9827580

1965516

1975343580

7864371

20604

31457484

47186226

15728742

162037500084

9. If you are to multiply any Number by an Unit with Cyphers, as by 10, 100, 1000, &c. then annex so many Cyphers before the Multiplicand, and that Number when the Cyphers are annexed is the Product required. As if you would multiply 428 by 100, annex two Cyphers to 428 and it is 42800. If it were required to multiply 102 by 10000, annex 4 Cyphers, and it gives 1020000 for the Product required.

### The Proof of Multiplication.

10. *Multiplication* is proved by *Division*, and to speak Truth, all other Ways are false (according to *Frisius*) and therefore it will be necessary in the first place to learn *Division*, and by that to prove *Multiplication*. There are some other Ways used indeed, but on a strict *Examen*, there is not one in a thousand of their Products right; therefore we omit them.

11. The general Effect of *Multiplication* is contained in the Definition of the same, which is to find out a third Number, so often containing one of the two given Numbers as the other containeth Units.

The second Effect is, by having the Length and Breadth of any Thing (as a Parallelogram or long Plain) to find the superficial Content of the same; and by having the superficial Content of the Base, and the Length, to find out the Solidity of any Parallelopipedon, Cylinder, or other solid Figure.

The Third Effect is, by the Contents, Price, Value, buying, selling, Expence, Wages, Exchange, Simple Interest, Gain or Loss of any one Thing, be it Money, Merchandize, &c. to find out the Value, Price, Expence, buying, selling, Exchange, or Interest, of any Number of Things of the like Name, Nature and Kind.

The fourth Effect is not much unlike the other, by the Contents, Value or Price of any one Part of any Thing denominated, to find the Contents, Value or Price of the whole Thing, all the Parts into which the Whole is divided, multiplying the Price of one of those Parts.

The fifth Effect is, to aid, to compound and to make other Rules, as chiefly, the *Rule of Proportion*, called the *Golden Rule*, or *Rule of Three*; also by it Things of one Denomination are reduced to another.

If

If you multiply any Number of Integers, by the Price of the Integer, the Product will discover the Price of the Quantity, or Number of Integers given.

In a rectangular Solid, if you multiply the Breadth of the Base by the Depth, and that Product by the Length, the last Product will discover the Solidity or Content of the same Solid.

*Some Questions proper to this Rule, may be these following.*

*Quest. 1.* What is the Content of a square Piece of Ground, whose Length is 28 Perches and Breadth 13?

*Answer,* 364 square Perches; for multiplying 28 the Length, by 13 the Breadth, the Product is so much.

*Quest. 2.* There is a square Battle, whose Flank is 47 Men, and the Files 19 deep, what Number of Men doth that Battle contain? *Facit* 893; for multiplying 47 by 19 the Product is 893.

*Quest. 3.* If any one Thing cost 4 Shillings, what shall 9 such Things cost? *Answ.* 36 Shillings; for multiplying 4 by 9 the Product is 36.

*Quest. 4.* If a Piece of Money or Merchandize be worth or cost 17 Shillings, what shall 19 such Pieces of Money or Merchandize cost? *Facit* 323 Shillings, which is equal to 16*l.* 3*s.*

*Quest. 5.* If a Soldier or Servant get or spend 14*s.* per Month, what is the Wages or Charges of 49 Soldiers or Servants for the same Time? Multiply 49 by 14, the Product is 686*s.* or 34*l.* 6*s.* for the Answer.

*Quest. 6.* If in a Day there are 24 Hours, how many Hours are there in a Year, accounting 365 Days to constitute the Year? *Facit* 8760 Hours, to which if you add the 6 Hours over and above 365 Days, as there is in a Year, then it will be 8766 Hours; now if you multiply this 8766 by 60, the Number of Minutes in an Hour, it will produce 525960, the Number of Minutes in a Year.

## C H A P. VII.

### *Division of Whole Numbers.*

1. **D**I**V**I**S**I**O**N is the separating or parting of any Number or Quantity given, into any Part assigned; or to find how often one Number is contained in another; or from any two Numbers given, to find a third that shall consist of so many Units, as the one of those two Numbers given is comprehended, or is contained in the other.

2. *Division* hath three Parts of Numbers remarkable, *viz.* first, the Dividend; 2*dly*, the Divisor, 3*dly*, the Quo-

tient



Divident. The Dividend is the Number given to be parted or divided. The Divisor is the Number given by which the Dividend is divided, or it is the Number which sheweth how many Parts the Dividend is to be divided into, and the Quotient is the Number produced by the Division of the two given Numbers the one by the other.

So 12 being given to be divided by 3, or into three equal Parts, the Quotient will be 4; for 3 is contained in 12 four times, where 12 is the Dividend, and 3 is the Divisor, and 4 is the Quotient

3. In *Division*, set down your Dividend, and draw a crooked Line at each End of it, and before the Line at the left Hand place the Divisor, and behind that on the right Hand place the Figures of the Quotient, as in the Margent, where it is required to divide 12 by 3;  $3)12(4$  first, I set down 12 the Dividend, and on each Side of it I draw a crooked Line, and before that on the left Hand do I place 3 the Divisor, then do I seek how often 3 is contained in 12, and because I find it four times, I put 4 behind the crooked Line, on the right Hand of the Dividend, denoting the Quotient.

4. But if, when the Divisor is a single Figure, the Dividend consisteth of two or more Places, then having placed them for the Work (as before directed) put a Point under the first Figure of the left Hand of the Dividend, provided it be bigger than (or equal to) the Divisor; but if it be lesser than the Divisor, then put a Point under the second Figure from the left Hand of the Dividend, which Figures, as far as the Point goeth from the left Hand, are to be reckoned by themselves, as if they had no Dependence upon the other part of the Dividend, and for Distinction sake may be called the Dividual; then ask how often the Divisor is contained in the Dividual, placing the Answer in the Quotient, then multiply the Divisor by the Figure that you placed in the Quotient, and set the Product thereof under your Dividual, then draw a Line under the Product, and subtract the said Product from the Dividual, placing the Remainder under the said Line; then put a Point under the next Figure in the Dividend, on the right Hand of that to which you put the Point before, and draw it down, placing it on the right Hand of the Remainder which you found by *Subtraction*, which Remainder, with the said Figure annexed before it, shall be a new Dividual; then seek again how often the Divisor is contained in this new Dividual, and put the Answer in the Quotient, on the right Hand of the Figure which you put there before; then multiply the Divisor by the last Figure that you put in the Quotient, and subscribe the



the Product under the Dividual and make Subtraction, and to the Remainder draw down the next Figure from the grand Dividend (having first put a Point under it) and put it on the right Hand of the Remainder for a new Dividual, as before : and proceed thus till the Work is finished.

Observing this general Rule in all Kinds of *Division*; first, to seek how often the Divisor is contained in the Dividual, then (having put the Answer in the Quotient) multiply the Divisor thereby, and subtract the Product from the Dividual: An Example or two will make the Rule Plain. Let it be required to divide 2184 by 6. I dispose of the Numbers

6) 2184 (3 given as is before directed, and as you see in the Margent; in order to the Work, then because 6 the Divisor is more than 2 the first Figure of the Dividend, I put a Point under 1 the second Figure, which makes 21 for the Dividual; then do I ask how often 6 the Divisor is contained in 21, and because I cannot have it more than 3 times, I put 3 in the Quotient, and thereby do I multiply the Divisor (6) and the Product is 18, which I set in order under the Dividual, and subtract it therefrom, and the Remainder (3) I place in order under the Line, as you see in the Margent.

$$\begin{array}{r}
 6 \overline{) 2184} \quad (3 \\
 \underline{18} \phantom{00} \\
 3 \phantom{00} \\
 6 \overline{) 2184} (36 \\
 \underline{18} \phantom{00} \\
 38 \phantom{00} \\
 \underline{36} \phantom{00} \\
 2 \phantom{00}
 \end{array}$$

Then do I make a Point under the next Figure of the Dividend, being 8, and draw it down, placing it before the Remainder 3, so have I 38 for a new Dividual; then do I seek how often 6 is contained in 38, and because I can't have it more than 6 times, I put 6 in the Quotient, and thereby do I multiply the Divisor 6, and the Product (36) I put under the Dividual (38) and subtract it therefrom, and the Remainder (2) I put under the Line, as you see in the Margent.

Then do I put a Point under the next (and last) Figure of the Dividend (being 4) and draw it down to the Remainder 2, and putting it on the right Hand thereof, it maketh 24 for a new Dividual; then I ask how often 6 is contained in 24, and the Answer is 4; which I put into the Quotient, and multiply the Divisor (6) thereby, and the Product (24) I put under the Dividual (24) and subtract it therefrom, and the Remainder is (0); and thus the Work is finished, and I find the Quotient to be 364, that is, 6 is contained in 2184 just 364 times, or 2184 being divided into 6 equal Parts, 364 is one of those Parts.

$$\begin{array}{r}
 6 \overline{) 2184} \quad (364 \\
 \underline{18} \phantom{00} \\
 38 \phantom{00} \\
 \underline{36} \phantom{00} \\
 24 \phantom{00} \\
 \underline{24} \phantom{00} \\
 0
 \end{array}$$

Again,

Again, if it were required to divide 2646 by 7, or into 7 equal Parts, the Quotient will be found to be 378, as by the following Operation appeareth.

$$7) 2646 \quad (378)$$

$$\begin{array}{r} \dots \\ 21 \\ \hline 54 \\ 49 \\ \hline 56 \\ 56 \\ \hline 00 \end{array}$$

So if it be required to divide 964 by 8, the Quotient will be found to be 118, and 2 remaining after Division is ended. The Work followeth :

$$8) 964 \quad (118)$$

*The work*

$$\begin{array}{r} 8 \\ \hline 8 \\ \hline 66 \\ 64 \\ \hline (2) \end{array}$$

Many times the Dividend cannot exactly be divided by the Divisor, but something will remain, as in the last Example, where 946 was given to be divided by 8, the Quotient was 118, and there remained 2 after the Division was ended : Now what is to be done in this Case with the Remainder, the Learner shall be taught when we come to treat of the reducing (or Reduction) of Fractions.

And here note, That if, after your Division is ended, any Thing do remain, it must be lesser than your Divisor, for otherwise your Work is not rightly performed.

Other Examples are such as follow.

$$8) 73464 \quad (9183)$$

$$\begin{array}{r} 72 \\ \hline 14 \\ 8 \\ \hline 66 \\ 64 \\ \hline 24 \\ 24 \\ \hline (0) \end{array}$$

$$9) 13758 \quad (1528)$$

$$\begin{array}{r} 9 \\ \hline 47 \\ 45 \\ \hline 25 \\ 18 \\ \hline 78 \\ 72 \\ \hline (6) \end{array}$$

But

5. But if the Divisor consisteth of more Places than one, then chuse so many Figures from the left Side of the Dividend for a Dividual as there are Figures in the Divisor, and put a Point under the farthest Figure of that Dividual to the right Hand, and seek how often the first Figure on the left Side of the Divisor is contained in the first Figure on the left Side of the Dividual, and place the Answer in the Quotient, and thereby multiply your Divisor, placing your Product under your Dividual, and subtract it therefrom, placing the remainder below the Line; then put a Point under the next Figure in the Dividend, and draw it down to the said Remainder, and annex it on the right Side thereof, which makes a new Dividual, and proceed as before, till the Work is finished.

And if it so happen, that after you have chosen your first Dividual, (as is before directed) you find it to be lesser than the Divisor, then put a Point under the Figure more near to the right Hand, and seek how often the first Figure on the left Side of the Divisor is contained in the two first Figures on the left Side of the Dividual, and place the Answer in the Quotient, by which multiply the Divisor, and place the Product thereof in order, under the Dividual, and subtract it therefrom, and proceed as before.

Always remembering that in all Cases of *Division*, if after you have multiplied your Divisor by the Figure last placed in the Quotient, if the Product be greater than the Dividual, then you must cancel that Figure in the Quotient, and instead thereof put a Figure lesser by an Unit (or one) and multiply the Divisor thereby, and if still the Product be greater than the Dividual, make the Figure in the Quotient yet lesser by an Unit, and thus do until your Product be lesser than the Dividual, or at the most equal thereto, and then make Subtraction, &c.

So if you would divide 9464 by 24, the Quotient will be found to be 394; I first put down the given Number, as is before directed in the 3d Rule.

24)9464(39  
 ...  
 72  
 ---  
 226  
 ---  
 216  
 ---  
 10

Now because my Divisor consisteth of two Figures, I therefore put a Point under the second Figure from the left Hand of my Dividend, which here is 4, wherefore I seek how often 2 the first Figure (on the left Side of the Divisor) is contained in 9, the like first in the Dividual, the Answer is 4, which I put in the Quotient, and thereby multiply all the Divisor, and find the Product to be 86, which is greater than the Dividual 94, wherefore I cancel the 4 in the Quotient, and instead thereof I put 3 (an

an Unit lesser) and by it multiply the Divisor 24, and the Product is 72, which I subtract from 94 the Dividual, and the Remainder is 22; then do I make a Point under the next Figure 6 in the Dividend, and draw it down and place it on the right Side of the Remainder 22, and it makes 226 for a new Dividual; now because the Dividual 226 consisteth of a Figure more than the Divisor, therefore I seek how often 2 (the first Figure of the Divisor) is contained in 22, the two first Figures of the Dividual, and I say 9 times, wherefore I put 9 in the Quotient, and thereby multiply the Divisor 24, the Product (216) I place under the Dividual 226, and subtract it, and there remaineth 10.

$$\begin{array}{r} 24) 9464 (39 \\ \dots \\ 72 \\ \hline 226 \\ 216 \\ \hline 10 \end{array}$$

Then I go on and make a Point under the next and last Figure (4) in the Dividend, and draw it down to the Remainder 10, and it makes 104 for a new Dividual, which is also a Figure more than the Divisor, and therefore I seek how often 2 is contained in 10, I answer 5 times; but multiplying my Divisor by 5, the Product is 120, which is greater than the Dividual, and therefore I make it but 4, and by it multiply the Divisor, and the Product is 96, which being placed under, and subtracted from the Dividual, there remaineth 8; and thus the whole Work of this Division is ended, and I find that 9464 being divided by 24, or into 24 equal Parts, is found to be 394, as was said before, and the Remainder is 8, as you see in the Work following.

$$24) 9464 (394$$

$$\begin{array}{r} 72 \\ \hline 226 \\ 216 \\ \hline 104 \\ 96 \\ \hline (8) \end{array}$$

Another Example may be this: Let there be required the Quotient of 1183653 divided by 385: First I dispose of the Numbers in order to their dividing, and because 118, the three first Figures of the Dividend, is lesser than the Divisor 385, I therefore make a Point under the fourth Figure, which is 3, and seek how often 3 (the first Figure of the Divisor) is contained in 11: the Answer is 3, which I put in the Quotient, and thereby multiply the Divisor 385, and the Product is 1155, which

$$\begin{array}{r} 385) 1183653 (3 \\ 1155 \\ \hline 28 \end{array}$$

which I subtract from the Dividual 1183, and there remain 28: Then, as before, I draw down the next Figure, which is 6, and place it before the Remainder 21, so have 1286 for a new Dividual, and because it hath no more Figures than the

Divisor, I seek how often 3 (the first Figure of the Divisor) is contained in 2 (the first Figure of the Dividual) and the Answer is 0; for a greater Number cannot be con-

tained in a lesser; wherefore I put 0 in the Quotient, and thereby, according to the fifth Rule, I should multiply the Divisor, but if I do the Product will be 0, and 0 subtracted from the Dividual 286, the Remainder is the same; wherefore I draw down the next Figure (5) from the Dividend, and put it before the said Remainder 286

so have 12865 for a new Dividual; and because it consisteth of four Places, with a Place more than the Divisor, I seek how often 3, the first Figure of the Divisor, is contained in 28, the two first of the Dividual, and I say there is 9 times 3 in 28

but multiplying my whole Divisor (385) thereby, I find the Product to be 3465, which is greater than the Dividual 2865; wherefore I chuse 8, which is lesser by an Unit than 9, and thereby I multiply my Divisor 385, and the Product is 3080, which still is greater than the said Dividual; wherefore I chuse another Number yet an Unit lesser, viz. 7, and having multiplied my Divisor thereby, the Product is 2695, which is lesser than the Dividual 2865, wherefore I put 7 in the Quotient, and subtract 2695 from the Dividual 2865, and there remains 170; then I draw down the last Figure (3) of the Dividend, and place it before the said Remainder 170, and it makes 1703 for a new Dividual; then, for the Reason

above said, I seek how often 3 is contained in 17 the Answer is 5, but multiplying the Divisor thereby, the Product is 1925, greater than the Dividual, wherefore I say it will bear 4 (an Unit lesser) and by it I multiply the Divisor 385 and the Product is 1540, which is lesser than the Dividual, and therefore I put 4 in the Quotient, and subtract the said Product from the Dividual, and there remains 163; and thus the Work is finish'd, and I find that 1183653 being divided by 385, or into 385 equal Shares or Parts, the Quotient or one of those Parts, is 3074, and besides there is 163 remaining.

385)1183653(3074  
 1155  
 2865  
 2695  
 1703  
 1540  
 163



And thus the Learner being well versed in the Method of the foregoing Examples, he may be sufficiently qualified for the dividing of any greater Sum or Number into as many Parts as he pleaseth; that is, he may understand the Method of dividing by a Divisor which consisteth of 4, or 5, or 6, or any greater Number of Places, the Method being the same with the foregoing Examples in every respect.

*Other Examples in Division.*

27986) 835684790 (29860

55972  
275964  
 251874  
240907  
 223888  
170199  
 167916

Remain (22830)

196374) 473986018 (2413

392748  
812380  
 785496  
268841  
 196374  
724678  
 589122

Remain (135556)

So if you divide 47386473 by 58736, you will find the Quotient to be 806, and 45257 will remain after the Work is ended.

In like manner; if you would divide 3846739204 by 483064, the Quotient will be 7963, and the Remainder after Division will be 100572.

*Compendiums in Division.*

1. IF any given Number be to be divided by another Number that hath Cyphers annexed on the right Side thereof, (omitting the Cyphers) you may cut off so many Figures from the right Hand of the Dividend, as there are Cyphers before the Divisor, and let the remaining Numbers in the Dividend be divided by the remaining Numbers

of the Divisor, observing this Caution, That if after your Division is ended any thing remain, you are to annex thereto the Number or Numbers that were cut off from the Dividend, and such new found Number shall be the Remainder. (See Mr. Doughtred's *Clavis Mathematica*, cap. 5. 3.) As for Example, Let it be required to divide 46658

4|90) 466|58 (116

$$\begin{array}{r}
 4 \\
 \hline
 6 \\
 4 \\
 \hline
 26 \\
 24 \\
 \hline
 (258)
 \end{array}$$

by 400; now because there are two Cyphers before the Divisor, I cut off as many Figures from before the Dividend, viz. 58, so that then there will remain only 466 to be divided by 4, and the Quotient will be 116, and there will remain 2, to which I annex the two Figures (58) which were cut off from the Dividend, and it makes 258 for the true Remainder; so that I conclude 46658 being divided by 400, the Quotient will be 116; and 258 remain after the Work is ended, as by the Work in the Margent.

2. And hence it followeth, that if the Divisor be 1, or an Unit with Cyphers annexed, you may cut off so many Figures from before the Dividend as there are Cyphers in the Divisor, and then the Figure or Figures that are on the left Hand will be the Quotient, and those that are on the right Hand will be the Remainder after the Division is ended. (*Vid. Gem. Fris. Arith. par. 1.*) As thus; if 45783 were to be divided by 10, I cut off the last Figure (3) with a Dash, thus, 4578|3, and the Work is done, and the Quotient is 4578, the Number on the left Hand of the Dash, and the Remainder is 3, on the right Hand. In the like manner, if the same Number 45683 were to be divided by 100, I cut off two Figures from the End, thus 457 83 and the Quotient is 457, and the Remainder is 83. And if I am to divide the same by 1000, I cut off three Figures from the End, thus, 45|783, and the Quotient is 45, and 783 is the Remainder, &c.

6. The general Effect of *Division* is contained in the Definition of the same, that is by having two unequal Numbers given, to find a third Number in such Proportion to the Dividend, as the Divisor hath to Unit or 1: It also discovers what Reason or Proportion there is between Numbers, so if you divide 12 by 4, it quotes 3, which shews the Reason or Proportion of 4 to 12 is triple.

The second Effect is, by the superficial Measure or Content, and the Length of any Oblong, Rectangular, Parallelogram, or square Plane known, to find out the Breadth thereby;

ereby; or contrarywise, by having the Superficies and breadth of the said Figure, to find out the Length thereof. Also by having the Solidity and Length of a Solid, to find the Superficies of the Base, & *contra*.

The third Effect is, by the Contents, Reason, Price, Value, Bying, Selling, Expences, Wages, Exchange, Interest, Profit, or Loss of any Number of Things, be it Money, Merchandize, or what else; to find out the Contents, Reason, Price, Value, Buying, Selling, Expence, Wages, Exchange, Interest, Profit or Loss of any one Thing of the same Kind.

The fourth Effect is, to aid, to compose and to make other Rules, but principally the Rule of Proportion, called the *Golden Rule*, or *Rule of Three*, and the Reduction of Monies, Weights and Measures of one Denomination into another; by it also Fractions are abbreviated, by finding a common Measure unto the Numerator and Denominator, thereby discovering commensurable Numbers.

If you divide the Value of any certain Quantity by the same Quantity, the Quotient discovers the Rate or Value of the Integer; as if 8 Yards of Cloth cost 96 Shillings, if you divide (96) the Value or Price of the given Quantity, by (8) the same Quantity, the Quotient will be 12, which is the Price or Value of one of those Yards.

If you divide the Value or Price of any unknown Quantity by the Value of the Integer, it gives you in the Quotient that unknown Quantity, whose Price is thus divided; as if 12 Shillings were the Value of a Yard, I would know how many Yards are worth 96 Shillings: Hence if you divide 96, the Price or Value of the unknown Quantity, by 12, the Rate of the Integer, 1 or 1 Yard, the Quotient will be 8, which is the Number of Yards worth 96s.

Some Questions answered by Division may be these following.

*Quest. 1.* If 22 Things cost 66 Shillings, what will 1 such Thing cost? *Facit* 3 Shillings; for if you divide 66 by 22, the Quotient is 3 for the Answer. So if 26 Yards or Ells of any Thing be bought or sold for 78<sup>l</sup>, how much will 1 Yard or Ell be bought or sold for? *Facit* 3<sup>l</sup>. for if you divide 78 by 26 Yards, the Quotient will be 3<sup>l</sup>. the Price of the Integer.

*Quest. 2.* If the Expence, Charges or Wages of 7 Years amount to 868<sup>l</sup>. what is the Expence, Charges or Wages of one Year? *Facit* 124<sup>l</sup>. for if you divide 868, the Wages of 7 Years, by 7, the Number of Years, the Quotient will be 124<sup>l</sup>. for the Answer. See the Work.

7) 868 (124

$$\begin{array}{r}
 7 \\
 \hline
 16 \\
 14 \\
 \hline
 28 \\
 28 \\
 \hline
 (0)
 \end{array}$$

*Quest. 3.* If the Content of one superficial Foot be 144 Inches, and the Breadth of a Board be 9 Inches, how many Inches of that Board in Length will make such a Foot? *Facit* 16 Inches; for by dividing 144, the Number of square Inches in a square Foot, by 9, the Inches in the Breadth of the Board, the Quotient is 16 for the Number of Inches in the Length of that Board to make a superficial Foot.

9) 144 (16 Inches

$$\begin{array}{r}
 9 \\
 \hline
 54 \\
 54 \\
 \hline
 (0)
 \end{array}$$

*Quest. 4.* If the Content of an Acre of Ground be 160 square Perches, and the Length of a Furlong (propounded) be 80 Perches, how many Perches will there go in Breadth to make an Acre? *Facit* 2 Perches; for if you divide 160 the Number of Perches in an Acre, by 80, the Length of the Furlong in Perches, the Quotient is 2 Perches, and so many in Breadth of that Furlong will make an Acre.

80) 160 (2 Perches,

$$\begin{array}{r}
 160 \\
 \hline
 (0)
 \end{array}$$

*Quest. 5.* If there be 893 Men to be made up into a Battalion, the Front consisting of 47 Men, what Number must there be in the File? *Facit* 19 deep in the File; for if you divide 893, the Number of Men, by 47, the Number in the Front, the Quotient will be 19 in Depth of the File. The Work followeth.

47) 893 (19 deep in File.

$$\begin{array}{r}
 47 \\
 \hline
 423 \\
 47 \\
 \hline
 (0)
 \end{array}$$

*Quest. 6.* There is a Table whose superficial Content is 72 Feet, and the Breadth of it at the End is 3 Feet; now I demand what is the Length of this Table? *Facit* 24 Feet long; for if you divide 72, the Content of the Table in Feet, by 3, the Breadth of it, the Quotient is 24 Feet for the Length thereof, which was required. See the Operation in the Margent.

$$\begin{array}{r} 3 \overline{) 72} \quad (24 \\ \underline{6} \phantom{0} \\ 12 \\ \underline{12} \\ (0) \end{array}$$

*The Proof of Multiplication and Division.*

*Multiplication and Division interchangeably prove each other; for if you would prove a Sum in Division, whether the Operation be right or no, multiply the Quotient by the Divisor, and if any thing remain after Division is ended add it to the Product, which Product, if your Sum was rightly divided, will be equal to the Dividend. And contrarywise, if you would prove a Sum in Multiplication, divide the Product by the Multiplier, and if the Work was rightly performed the Quotient will be equal to the Multiplicand. See the Example, where the Work is done and undone. Let 7654 be given to be multiplied by 3242, the Product will be 24814268, as by the Work appeareth.*

$$\begin{array}{r} 7654 \\ \times 3242 \\ \hline 15308 \\ 30616 \\ 15308 \\ 22962 \\ \hline 24814268 \end{array}$$

And then if you divide the said Product 24814268 by 3242 the Multiplier, the Quotient will be 7654, equal to the given Multiplicand.

$$3242 \overline{) 24814268} \quad (7654$$

$$\begin{array}{r} 22694 \\ \times 21202 \\ \hline 19452 \\ 17506 \\ 16210 \\ \hline 12968 \\ 12968 \\ \hline (0) \end{array}$$

In like manner (to prove a Sum or Number in *Division*) if 24814268 were divided by 3242, the Quotient will be found to be 7654; then for Proof, if you multiply 7654 the Quotient, by 3242 the Divisor, the Product will amount to 24814268, equal to the Dividend

Or, you may prove the last, or any other Example in *Multiplication*, thus, viz. divide the Product by the Multiplicand, and the Quotient will be equal to the Multiplier. See the Work.

$$7654$$



$$\begin{array}{r}
 7654 \\
 3242 \\
 \hline
 15308 \\
 30616 \\
 15308 \\
 \hline
 22962 \\
 7654 \overline{) 24814268} (3242 \\
 \dots
 \end{array}$$

$$\begin{array}{r}
 22962 \\
 \hline
 18522 \\
 15308 \\
 \hline
 32146 \\
 30616 \\
 \hline
 15308 \\
 15308 \\
 \hline
 (0)
 \end{array}$$

From whence there arises this Corollary, that any Operation in *Division* may be proved by *Division*; for if, after your *Division* is ended, you divide the Dividend by the Quotient, the new Quotient thence arising will be equal to the Divisor of the first Operation; for Tryal whereof see the last Example be again repeated.

$$3242 \overline{) 24814268} (7654$$

$$\begin{array}{r}
 22694 \\
 \hline
 21202 \\
 19452 \\
 \hline
 17506 \\
 16210 \\
 \hline
 12968 \\
 12968 \\
 \hline
 (0)
 \end{array}$$

For Proof whereof divide again 24814268 by the Quotient 7654, and the Quotient hence will be equal to the first Divisor 3242. See the Work.

$$7654 \overline{) 24814268} (3242$$

$$\begin{array}{r}
 22962 \\
 \hline
 18522 \\
 15308 \\
 \hline
 32146 \\
 30616 \\
 \hline
 15308 \\
 15308 \\
 \hline
 (0)
 \end{array}$$

But in proving Division by Division, the Learner is to observe this following Caution; That if after his Division is ended, there be any Remainder, before you go about to prove your Work, subtract the Remainder out of your Dividend, and then work as in the following Example, where it is required to divide 43876 by 765; the Quotient here is 57, and the Remainder is 271. See the Work following.

$$765) 43876 \text{ (57)}$$

$$\begin{array}{r} 3825 \\ \underline{5626} \\ 5355 \\ \underline{(271)} \end{array}$$

Now to prove this Work, subtract the Remainder 271 out of the Dividend 43876; and there remaineth 43605, for a new Dividend to be divided by the former Quotient 57, and the Quotient thence arising is 765, equal to the given Divisor, which proveth the Operation to be right.

$$\begin{array}{r} 43876 \\ \underline{271} \\ 57) 43605 \text{ (765)} \end{array}$$

$$\begin{array}{r} 399 \\ \underline{370} \\ 342 \\ \underline{285} \\ 285 \\ \underline{(0)} \end{array}$$

Thus we have gone through the four Species of *Arithmetick*, viz. *Addition*, *Subtraction*, *Multiplication* and *Division*, upon which all the following Rules, and all other Operations whatsoever that are possible to be wrought by Numbers, have their immediate Dependance, and by them are resolved. (Vide *Gem. Fris. Arith. par. 1.*) Therefore before the Learner make a farther Step in this Art. let him be well acquainted with what has been delivered in the foregoing Chapter.

## C H A P. VIII.

## Of Reduction.

1. **R E D U C T I O N** is that which brings together two or more Numbers of different Denominations into one Denomination ; (*Hill's Arith. c. 13.*) or it serveth to change or alter Numbers, Money, Weight, Measure or Time, from one Denomination to another ; and likewise, to a-bridge Fractions to the lowest Terms : All which it doth to precisely, that the first Proportion remaineth without the least Jot of Error or Wrong committed ; so that it be- longerth as well to Fractions as Integers, of which in the proper Place, *Reduction* is generally performed either by *Multiplication* or *Division* ; from whence we may gather, That

2. *Reduction* is either descending or ascending.

3. *Reduction* descending, is when it is required to reduce a Sum or Number of a greater Denomination into a lesser ; which Number, when it is so reduced, shall be equal in Value to the Number first given in the greater Denomina- tion ; (*Wing. Arith. 7, 2, 3, 4.*) as if it were required to know how many Shillings, Pence or Farthings are equal in Value to 200*l.* or how many Ounces are contained in 45 C. Weight ; or how many Days, Hours or Minutes there are in 240 Years, &c. And this Kind of *Reduction* is generally performed by *Multiplication*.

4. *Reduction* ascending, is when it is repuired to reduce or bring a Sum or Number of a smaller Denomination into a greater, which shall be equivalent to the given Number ; as suppose it were required to find out how many Pounds, Shillings or Pence are equal in Value to 43785 Farthings ; or how many Hundreds are equal to, or in 3748 Pounds, &c. And this Kind of *Reduction* is always performed by *Division*.

5. When any Sum or Number is given to be reduced into another Denomination, you are to consider whether it ought to be resolved by the Rule descending, or a- scending, &c by *Multiplication* or *Division* : If it be to be performed by *Multiplication*, consider how many Parts of the Denomination into which you would reduce it are contain'd in an Unit or Integer of the given Number, and multiply the said given Number thereby, and the Product thereof will be the Answer to the Question. As if the Question were, In 38 Pounds how many Shillings ? Here I consider,

consider, that in a Pound are 20 Shillings, and that the Number of Shillings in 38l. will be 20 times 38, wherefore I multiply 38l. by 20, and the Product is 760, and so many Shillings are contained in 38l. as in the Margent.

But when there is a Denomination or Denominations between the Number given and the Number required, you may, if you please, reduce it into the next inferior Denomination, and then into the next lower than that, &c. untill you have brought it into the Denomination required. As for Example, Let it be demanded in 132 Pounds how many Farthings? First, I multiply 132, the Number of Pounds given, by 20, to bring it into Shillings, and it makes 2640 Shillings, then do I multiply the 2640 Shillings by 12, to bring them into Pence, and it produceth 31680, and so many Pence are contained in 2640 Shillings, or 132 Pounds; then do I multiply the Pence, viz. 31680 by 4, to bring them into Farthings (because 4 Farthings is a Penny) and I find the Product thereof to be 126720, and so many Farthings are equal in Value to 132 Pounds. As by the Work in the Margent.

6. And if the Number propounded to be reduced is to be divided, or wrought by the Rule ascending, consider how many of the given Number are equal to an Unit or Integer, in that Denomination to which you would reduce your given Number, and make that your Divisor, and the given Number your Dividend: and the Quotient thence arising will be the Number sought or required. As for Example, Let it be required to reduce 2640 Shillings into Pounds. Here I consider that 20 Shillings are equal to one Pound, wherefore I divide 2640, the given Number, by 20, and the Quotient is 132; and so many Pounds are contained in 2640 Shillings. In *Reduction* descending and ascending, the Learner is advised to take particular Notice of the Tables delivered in the second Chapter of this Boook, where he may be informed what Multipliers and Divisors to make use of in the reducing of any Number to any other Denomination whatsoever, especially *English* Money, Weights, Measures, Time, and Motion: But in this Place it is not convenient to meddle with foreign Coins, Weights or Measures.

$$\begin{array}{r}
 132 \text{ Pounds} \\
 \underline{20} \\
 2640 \text{ Shil.} \\
 \underline{12} \\
 5280 \\
 \underline{2640} \\
 31680 \text{ Pence} \\
 \underline{4} \\
 126720 \text{ Farth.}
 \end{array}$$

$$\begin{array}{r}
 2 \overline{) 2640} (132 \\
 \underline{2} \phantom{00} \\
 4 \phantom{00} \\
 \underline{4} \phantom{00} \\
 0 \phantom{00} \\
 \underline{0} \phantom{00} \\
 0 \phantom{00} \\
 \underline{0} \phantom{00} \\
 0
 \end{array}$$

But if, in *Reduction* ascending, it happens that there is a Denomination or Denominations between the Number given and the Number required, then you may reduce your Number given into the next superior Denomination, and when it is so reduced, bring it into the next above that, and so on until you have brought it into the Denomination required. As for Example, Let it be demanded in 126720 Farthings how many Pounds? First, I divide my given Number, being Farthings, by 4, to bring them into Pence, because 4 Farthings make one penny, and there are 31680 Pence; then I divide 31680 Pence by 12, and the Quotient giveth 2640 Shillings; and then I divide 2640 Shillings by 20, and the Quotient giveth 132 Pounds, which are equal in Value to 126720 Farthings. See the whole Work as it followeth.

$$\begin{array}{r}
 \begin{array}{r}
 12 \overline{) 126720} \\
 \underline{12} \phantom{00} \\
 6 \phantom{00} \\
 \underline{4} \phantom{00} \\
 27 \phantom{00} \\
 \underline{24} \phantom{00} \\
 32 \phantom{00} \\
 \underline{32} \\
 (0)
 \end{array}
 \quad
 \begin{array}{r}
 12 \overline{) 31680} \\
 \underline{12} \phantom{00} \\
 24 \phantom{00} \\
 \underline{24} \phantom{00} \\
 48 \phantom{00} \\
 \underline{48} \phantom{00} \\
 (0)
 \end{array}
 \quad
 \begin{array}{r}
 20 \overline{) 2640} \\
 \underline{20} \phantom{00} \\
 6 \phantom{00} \\
 \underline{6} \phantom{00} \\
 4 \phantom{00} \\
 \underline{4} \phantom{00} \\
 (0)
 \end{array}
 \quad
 \begin{array}{r}
 1. \overline{) 132} \\
 \underline{1} \phantom{00} \\
 2 \phantom{00} \\
 \underline{2} \phantom{00} \\
 0
 \end{array}
 \end{array}$$

7. When the Number given to be reduced consisteth of divers Denominations, as *Pounds, Shillings, Pence* and *Farthings*; or of *Hundreds, Quarters, Pounds* and *Ounces, &c.* then you are to reduce the highest, or greatest, Denomination into the next inferior, and add thereunto the Number standing in the Denomination which your greatest or highest Number is reduced to; then reduce that Sum into the next inferior Denomination, adding thereto the Number standing in that Denomination; do so until you have brought the Number given into the Denomination proposed.

*As if it were required to reduce 48l. 13s. 10d. into Pence:* First, I bring 48l. into Shillings, by multiplying it by 20, and the Product is 960 Shillings, to which I add the 13 Shillings, and they make 973; then I multiply 973 by 12,

$$\begin{array}{r}
 \begin{array}{r}
 48 \text{ l. } 13 \text{ s. } 10 \text{ d.} \\
 \underline{20} \\
 960 \text{ Shil.} \\
 \text{Add } 13 \\
 \hline
 \text{Sum } 973 \\
 \underline{12} \\
 1946 \\
 \underline{973} \\
 11676 \text{ Pence} \\
 \text{Add } 10 \\
 \hline
 \text{Sum } 11686
 \end{array}
 \end{array}$$



to bring the Shillings into Pence, and they make 11676 Pence, to which I add the 10d. and they make 11686 Pence for the Answer. See the Work done.

8. If, in *Reduction* ascending, after Division is ended, any thing remain, such Remainder is of the same Denomination with the Dividend.

*Example.* In 4783 Farthings, I demand how many Pounds?

First, I divide the given Number of Farthings, viz. 4783. by 4, to bring them into Pence, and the Quotient is 1195 Pence, and there remaineth 3 after the Work of Division is ended, which is 3 Farthings.

Again; I divide 1195 Pence (the said Quotient) by 12. to reduce them into Shillings, and the Quotient is 99 Shillings, and there is a Remainder of 7, which is 7 Pence.

And then I divide 99 Shillings (the last Quotient) by 20, to bring it into Pounds, and the Quotient is 4. and there remaineth 19 Shillings; so I conclude that in 4783, the proposed Number of Farthings, there is 4 Pounds, 19 Shillings, 7 Pence, 3 Farthings. View the following Operation.

$$\begin{array}{r}
 \begin{array}{r}
 12 \overline{) 4783} \\
 \underline{48} \phantom{00} \\
 78 \phantom{00} \\
 \underline{72} \phantom{00} \\
 63 \phantom{00} \\
 \underline{60} \phantom{00} \\
 33 \phantom{00} \\
 \underline{32} \phantom{00} \\
 13
 \end{array}
 \end{array}$$

Rem. (3) Farthings.

*More Examples in Reduction of Coin:*

*Quest. 1.* In 438l. how many Shillings?

*Facit* 8760 Shillings; for by multiplying the 438 by 20, the Product amounteth to so much. See the Work in the Margent.

$$\begin{array}{r}
 438 \text{ l.} \\
 \underline{20} \\
 8760 \text{ s.}
 \end{array}$$

*Quest. 2.* In 467l. how many Pence? First, multiply the given Number of Pounds (467) by 20, to bring it into Shillings, and it makes 9340 Shillings; then multiply the Shillings by 12, and it produceth 112080 Pence, as in the Margent.

$$\begin{array}{r}
 467 \text{ Pounds} \\
 \underline{20} \\
 9340 \text{ Shil.} \\
 \underline{12} \\
 112080 \\
 \underline{9340} \\
 112080 \text{ Pence}
 \end{array}$$

*Facit* 112080 Pence

467 Pounds Or it may be resolved thus, viz.  
 $\begin{array}{r} 467 \text{ Pounds} \\ 240 \\ \hline 18680 \\ 934 \\ \hline \text{Facit } 112080 \text{ Pence} \end{array}$  multiply the given Number of Pounds  
 467, by 240, the Number of Pence  
 in a Pound, and the Product is the  
 same, viz. 112080 Pence as by the  
 Operation appeareth.

Quest. 3. In 5673<sup>l</sup>. how many Farthings? First, multiply  
 the given Number by 20, to bring it into Shillings, and it  
 produceth 113460 Shillings; then multiply that Product by  
 12, to bring it into Pence, and it produceth 1361520 Pence;  
 then, lastly, multiply the Pence by 4, and it produceth  
 5446080 Farthings.

$\begin{array}{r} 5673 \text{ Pounds} \\ 20 \\ \hline 113460 \text{ Shillings} \\ 12 \\ \hline 226920 \\ 113460 \\ \hline 1361520 \text{ Pence} \\ 4 \\ \hline \text{Facit } 5446080 \text{ Farthings} \end{array}$

Or this Question might have been thus resolved, viz.  
 multiply 5673, the given Number of the Pounds, by 960,  
 the Number of Farthings in a Pound, and it produceth  
 the same Effect, as you may see by the Work.

$\begin{array}{r} 5673 \text{ Pounds} \\ 960 \\ \hline 340380 \\ 51057 \\ \hline \text{Facit } 5446080 \text{ Farthings} \end{array}$   $\begin{array}{r} 20 \text{ Shillings} \\ 12 \\ \hline 240 \text{ Pence} \\ 4 \\ \hline 960 \text{ Farthings} \end{array}$

Otherwise thus: First bring the given Number 5673<sup>l</sup>.  
 into Shillings, and multiply the Shillings by 48, the Num-  
 ber of Farthings in a Shilling, and they same Effect is there-  
 by likewise produced, viz.

$\begin{array}{r} 5673 \text{ Pounds} \\ 20 \\ \hline 113460 \text{ Shillings} \\ 48 \\ \hline 907680 \\ 453840 \\ \hline \text{Facit } 5446080 \end{array}$   $\begin{array}{r} 12 \text{ Pence} \\ 4 \\ \hline 48 \end{array}$

These various Ways of Operation are expressed to inform  
 the Judgment of the Learner with the Reason of the Rule.  
 More Ways may be shewn, but these are sufficient even for  
 the meanest Capacities.

Quest.

8.  
iz.  
ds  
ce  
he  
he  
ly  
it  
by  
ce;  
th  
  
iz.  
so,  
th  
  
3l.  
m-  
re-  
  
rm  
le.  
for  
est,

# hap. 8.

## Reduction.

67

Quest. 4. In 458l. 16s. 7d. 3grs. how many Farthings?  
To resolve this Question, consider the 7th Rule of this  
Chapter, and work as you are there directed, and you will  
find the aforesaid given Number to amount to 440479 Far-  
things, viz.

	l.	s.	d.	grs.
	458	16	7	3
	20			
	<hr/>			
	9160			
Add		16	Shillings	
Sum	9176		Shillings	
	12			
	<hr/>			
	18352			
	9176			
	<hr/>			
	110112		Pence	
Add		7		
Sum	110119			
	4			
	<hr/>			
	440476		Farthings	
Add	13			
Sum	440479		Farthings	

This last Question, or any other of this Kind, may be  
more concisely resolved thus, viz. When you multiply the  
Pounds by 20, to bring them into Shillings, to the Product  
of the first Figure add the Figure standing in the place of  
Units in the Denomination of Shillings; but because the  
first Figure in the Multiplier is 0, I say, 0 times 8 is nothing,  
but 6 is 6, which I put down for the first Figure in the Pro-  
duct, then because the Multiplier is 0, I go on no further  
with it for if I should the whole Product will be 0, but pro-  
ceed, and when I come to multiply by the second Figure  
in the Multiplier, to the Product of it I add the Figure  
standing in the place of Tens in the Denomination of Shil-  
lings, which is 1, saying, 2 times 8 is 16, and the said Fi-  
gure 1 is 17; then I set down 7, and carry the Unit to the  
Product of the next Figure, as is directed in the 5th Rule of  
the 6th Chapter foregoing, and finish the Work; so that  
now you may have the whole Product and Sum of Shillings  
at one Operation, which is the same as before; and when  
you multiply the Shillings by 12, to bring them into Pence  
(after the same manner) add to the Product the Number  
standing in the Denomination of Pence, and so when you  
multiply the Pence by 4, to bring them into Farthings, add  
to the Product the Number standing under the Denomina-  
tion of Farthings. See the last Question thus wrought.

$$\begin{array}{r}
 \text{l. s. d. grs.} \\
 458 \quad 16 \quad 7 \quad 3 \\
 20 \\
 \hline
 9176 \text{ Shillings} \\
 12 \\
 \hline
 18359 \\
 9176 \\
 \hline
 110119 \\
 4 \\
 \hline
 \text{Facit } 440479 \text{ Farthings.}
 \end{array}$$

After the Method last prescribed are all the following Examples, that are of the same Nature, wrought and resolved.

*Quest. 5.* In 4375866 Farthings, I demand how many Pounds, Shillings and Pence?

To resolve this Question, First, I divide the given Number of Farthings by 4, and Quotient is 1093966 Pence and there remaineth 2 after the Division is ended, which (by the 8th Rule foregoing) is two Farthings; then I divide 1093966 Pence by 12, and the Quotient is 91163 Shillings and there remaineth 10 after Division, which, by the 8th Rule is so many Pence, viz. 10d. then I divide 91163 Shillings by 20, and the Quotient is 4558l. and there remaineth 3 Shillings, so the Work is finish'd and I find that in 4375866 Farthings, there are 4558l. 3s. 10d. 2grs. See the Operation.

$$\begin{array}{r}
 \text{12} \quad \text{20} \quad \text{l.} \\
 4 \overline{) 4375866} \quad 1093966 \quad 91163 \quad 4558
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{r}
 4 \\
 \hline
 37 \\
 36 \\
 \hline
 15 \\
 12 \\
 \hline
 38 \\
 36 \\
 \hline
 26 \\
 24 \\
 \hline
 26 \\
 24 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 108 \\
 \hline
 13 \\
 12 \\
 \hline
 19 \\
 12 \\
 \hline
 76 \\
 72 \\
 \hline
 46 \\
 36 \\
 \hline
 (10) d.
 \end{array}
 \quad
 \begin{array}{r}
 8 \\
 \hline
 11 \\
 10 \\
 \hline
 11 \\
 10 \\
 \hline
 16 \\
 16 \\
 \hline
 13 s.
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (2) \text{ grs.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \quad \text{grs.} \\
 \text{Facit } 4558 \quad 3 \quad 10 \quad 2
 \end{array}$$

*Quest.*

*Quest. 6.* In 4386<sup>l</sup>. I demand how many Groats?  
To resolve this Question, I reduce the given Number of  
pounds into Shillings, and they are 87720 Shillings; now  
consider that in a Shilling are 3 Groats, therefore I mul-  
tiply the Shillings by 3, and it produceth 263160 Groats.  
see the Work.

Facit 263160 Groats

Thus Question might have been otherwise resolved thus, viz. considering that in a Pound (or 20 Shillings) there are three times 20 Groats, which makes 60, by which I multiply the Number of Pounds given, and it produceth the same Effect at one Operation, as followeth.

Facit 263160 Groats in 4386 l.

Quest. 7 In 43758 Three-pences, I desire to know how many Pounds?

To resolve this, and many such like Questions, First, I divide my given Number of Three-pences by 4, because 4 Three-pences are in a Shilling, and the Quotient is 10939 Shillings, and there remaineth 2 after Division is ended, which is 2 Three-pences (by the 8th Rule of this Chapter) which are equal in Value to 6d. then I divide 10939 Shillings by 20, and the Quotient giveth 546l. and 19s. remains; so that I conclude in 43758 Pieces, of Three-pence per piece, there are 546l. 19s. 6d. as by the Work appear-  
eth.

$  \begin{array}{r}  4 \\  \hline  37 \\  36 \\  \hline  15 \\  12 \\  \hline  38 \\  36 \\  \hline  (2)  \end{array}  $	$  \begin{array}{r}  10 \\  \hline  9 \\  8 \\  \hline  13 \\  12 \\  \hline  19  \end{array}  $
	Shillings
	pences

**This**



The Question might have been otherwise resolved *viz.* First, multiply the given Number of Three-pence 43758, by 3, the Number of Pence in Three-pence, and the Product, *viz.* 131274 is the Number of Pence equal to the given Number of Three-pences, which Number of Pence may be brought into Pounds by dividing by 12, and by 20, and the Quotient you will find to be equal to the former Work, 546*l.* 19*s.* 6*d.*

$$\begin{array}{r}
 43758 \\
 \times 3 \\
 \hline
 131274
 \end{array}
 \quad
 \begin{array}{r}
 20 \\
 10939
 \end{array}
 \quad
 \begin{array}{r}
 l. \\
 546
 \end{array}
 \quad
 \begin{array}{r}
 s. \\
 19
 \end{array}
 \quad
 \begin{array}{r}
 d. \\
 6
 \end{array}$$
  

$$\begin{array}{r}
 12 \overline{) 112} \\
 \underline{108} \\
 47 \\
 \underline{36} \\
 114 \text{ re. (19) Shillings} \\
 \underline{108}
 \end{array}
 \quad
 \begin{array}{r}
 10 \overline{) 9} \\
 \underline{8} \\
 13 \\
 \underline{12}
 \end{array}$$

(6) pence remains

Or thus, Divide the given Number of Three-pences by the Number of Three-pences in a Pound, or 20 Shilling (which you will find to be 80, if you multiply 20*s.* by the Number of Three-pences in a shilling,) and you will find the Quotient to be 546*l.* as before, and a Remainder of 78 Three-pences; and if you divide those 78 Three-pences by 4, because there are 4 Three-pences in a Shilling, you will find the Quotient to be 19*s.* and 2 Three-pences remain, which are equal to 6*d.* which is the same that was before found.

$$\begin{array}{r}
 80 \overline{) 43758} \\
 \underline{40} \\
 37 \\
 \underline{32} \\
 55 \\
 \underline{48} \\
 4) 78 \text{ (19s.)} \\
 \underline{4} \\
 38 \\
 \underline{36} \\
 (2) \text{ Three-pences, or } 6d.
 \end{array}
 \quad
 \begin{array}{r}
 20 \\
 4 \\
 \hline
 80
 \end{array}$$

rap.  
ed the  
ence,  
ce, a  
qual  
ber  
12, a  
to t

Quest. 8. In 4785*l.* 13*s.* how many Pieces of 13*d.*  $\frac{1}{2}$  or Piece?

This Question cannot be resolved by Reduction descending or ascending absolutely, because 13*d.*  $\frac{1}{2}$  is no even part of a Pound, but rather by them jointly, *viz.* by Multiplication and Division; but if you bring the Number given into Half-pence, and divide the Half-pence by the Half-pence in 13*d.*  $\frac{1}{2}$  *viz.* 27, the Quotient will be the Answer: For having brought 4785*l.* 13*s.* into Half-pence, I find it makes 2297112, which I divide by 27, because there are so many Half-pence in 13*d.*  $\frac{1}{2}$ , and the Quote gives 85078 Pieces of 13*d.*  $\frac{1}{2}$ , and 6 Half pence remain over and above. Observe the Work following.

$\begin{array}{r} l. \\ 4785 \\ 20 \end{array}$	$\begin{array}{r} s. \\ 13 \end{array}$	$\begin{array}{r} d. \\ 13 \frac{1}{2} \\ 2 \end{array}$
---	---	--

95713 Shillings                      27 Half-pence  
24 Half-pence in a Shilling

$$\begin{array}{r} 382852 \\ 191426 \end{array}$$

2297112 Half-pence is the given Number

27)2297112 (85078 Pieces of 13*d.*  $\frac{1}{2}$

216

137

135

211

189

222

216

Remain (6) Half-pence

It would have produced the same Answer, if you had reduced your given Number into Farthings, and divided by the Farthings in 13*d.*  $\frac{1}{2}$ , *viz.* 54, (for always the Dividend and the Divisor must be of one Denomination) and then

then you would have had a Remainder of 12 Farthings which are equal in Value to the former Remainder of Half-pence, as you may prove at your Leisure.

*Quest. 9.* In 540 Dollars, at 4s. 4d. per Dollar, how many Pounds sterling?

First, bring your given Number of Dollars into Pence and then your Pence into Pounds, according to the former Directions, thus, in 4s. 4d. viz. a Dollar, you will find 52 Pence, by which multiply 540 Dollars, and it produceth 28080 Pence, which if you divide by 240, the Pence in one Pound, the Quotient will give you 117l. which is equal in Value to 540 Dollars, at 4s. 4d. per Dollar.

$$\begin{array}{r}
 \begin{array}{r}
 540 \\
 \times 52 \\
 \hline
 1080 \\
 2700 \\
 \hline
 28080
 \end{array}
 \end{array}$$

240) 28080 (117

$$\begin{array}{r}
 24 \\
 \times 40 \\
 \hline
 960 \\
 \times 24 \\
 \hline
 1680 \\
 1680 \\
 \hline
 0
 \end{array}$$

The foregoing Question might have been otherwise wrought thus, viz. multiply 540, your given Number of Dollars, by 13 the Number of Groats in a Dollar, or 4s. 4d. and it produceth 7020 Groats, which divide by 60, the Groats in one Pound, or 20 Shillings and the Quotient is 117, as before. See the Work.

$$\begin{array}{r}
 \begin{array}{r}
 540 \\
 \times 13 \\
 \hline
 1620 \\
 540 \\
 \hline
 7020
 \end{array}
 \end{array}$$

60) 7020 (117

$$\begin{array}{r}
 6 \\
 \times 10 \\
 \hline
 60 \\
 \times 6 \\
 \hline
 420 \\
 420 \\
 \hline
 0
 \end{array}$$

*Quest. 10.* In 547386 Pieces of 4d.  $\frac{1}{2}$  per Piece, I demand how many Pounds, Shillings and Pence?

First bring your given Number of Four-pence Half-penny all into Half-pence, which you will do if you multiply by 9, the Number of Half pence in 4d.  $\frac{1}{2}$ , and the Product is 4926474 Half-pence, which are brought into Pounds, if you divide them by 24, the Half-pence in a Shilling, and 20, the Shillings in a Pound, it makes 10263*l.* 9*s.* 9*d.*

$$\begin{array}{r}
 547386 \\
 \times 9 \\
 \hline
 4926474
 \end{array}
 \quad
 \begin{array}{r}
 2 \overline{) 0} \\
 \hline
 20526 \overline{) 9} \quad (10263
 \end{array}
 \quad
 \begin{array}{r}
 d \\
 4 \frac{1}{2} \\
 2 \\
 \hline
 9 \text{ Half-pence}
 \end{array}$$

$$\begin{array}{r}
 48 \\
 \hline
 126 \\
 120 \\
 \hline
 64 \\
 48 \\
 \hline
 167 \\
 144 \\
 \hline
 234 \text{ rem. (9) Shillings} \\
 216 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 20 \\
 \hline
 05 \\
 4 \\
 \hline
 12 \\
 12 \\
 \hline
 6 \\
 6 \\
 \hline
 \end{array}$$

*Facts* 10263 9 9

Remains (18) Half-pence, or 9*d.*

*Quest. 11.* In 4368*l.* I demand how many Pieces of 6*d.* of 4*d.* and of 2*d.* of each an equal Number? That is to say, What Number of Six-pences, Groats and Two-pences will make 4368*l.* and the Number of each equal?

The Way to resolve Questions of this Nature, is to add the several Pieces into which the given Number is to be brought into one Sum, and reduce the given Number into the same Denomination with their Sum, and to divide the said given Number so reduced by the said Sum, and the Quotient will give you the exact Number of each Piece: And after the same Method will we proceed to resolve the present Question, *viz.*

$$\begin{array}{r}
 4386 \text{ Pounds} \\
 240 \text{ Pence} \\
 \hline
 175440 \\
 8772 \\
 \hline
 12) 1052640 (87720 \\
 \dots
 \end{array}$$

Sum 12

$$\begin{array}{r}
 96 \\
 \hline
 92 \\
 84 \\
 \hline
 86 \\
 84 \\
 \hline
 24 \\
 24 \\
 \hline
 (0)
 \end{array}$$

Facit 87720 Pieces of 6 4

So that I conclude by the Operation, that 87720 Sixpences, and 87720 Groats, and 87720 Two-pences, are just as much, or equal to 4386*l.* or if you admit of 5*l.* to be thus divided, it is equal to 5 Sixpences, and 5 Fourpences or Groats, and 5 Two-pences.

Another Question of the same Nature with the last may be this following, *viz.*

*Quest. 12.* A Merchant is desirous to change 148*l.* into Pieces of 13*d.*  $\frac{1}{2}$ , of 12*d.* of 9*d.* of 6*d.* and of 4*d.* and he will have of each sort an equal Number of Pieces, I desire to know the Number?

Do as you were taught in the last Question, *viz.* add the several Pieces together, and reduce the Sum into Half-pence; then reduce the Sum to be changed, *viz.* 148*l.* into the same Denomination, and divide the greater by the lesser, and in the Quotient you will find the Answer, *viz.* 798, which is the Number of each of the Pieces required, and 18 remaineth, which is 18 Half-pence, by the 8th Rule of this Chapter. See the Work as followeth.



6d.  
4d.  
2d.  
12d.

l.  
148  
240  

---

5920  
296  

---

35520 Pence in 148l.  
2

71040 Half-pence

89) 71040 (798 Pieces of each Sort

623

874  
801

730  
712

d.  
13  $\frac{1}{2}$   
12  
9  
6  
4

Sum 44  $\frac{1}{2}$   
2

89 Halfp.

Rem. (18) Half-pence.

The Truth of the two foregoing Operations will thus be proved, *viz.* multiply the Answer by the Parts or Pieces into which the given Number was reduced, and having added the several Products together, if their Sum be equal to the given Number the Answer is right, otherwise not; so the Answer to the 11th Question was 87720, which is proved as followeth, *viz.*

l.  
87720 { Six-pences make 2193  
Four-pences make 1462  
Two-pences make 731

The total Sum of them 4386 which was the Sum given to be changed.

The Answer to the 12th Question was 798, and 18 Half-pence remained after the Work was ended; now the Truth of the Work may be proved as the former, *viz.*

		<i>l.</i>	<i>s.</i>	<i>d.</i>
798	Pieces of 13 <i>d.</i> $\frac{1}{2}$ make	44	17	9
	Pieces of 12 make	39	18	0
	Pieces of 9 make	29	18	6
	Pieces of 6 make	19	19	0
	Pieces of 4 make	13	06	0
and 18 Half-pence, or 9 <i>d.</i> remain		00	00	9

The total Sum of them 148 00 0  
 which total Sum is equal to the Number that was first  
 given to be changed, and therefore the Operation was  
 rightly performed.

### Reduction of Troy-weight.

We come now to give the Learner a few Examples in  
 Troy-weight; in working whereof he must be mindful  
 of the Table of Troy-weight delivered in the second  
 Chapter of this Book.

*Quest.* 13. In 482*lb.* 7 *oz.* 13 *pwt.* 21 *gr.* how many  
 Grains?

<i>lb.</i>	<i>oz.</i>	<i>pwt.</i>	<i>gr.</i>
482	7	13	21
<hr/>			
		12	
		971	
		482	
		<hr/>	
		5791	Ounces
		20	
		115833	Penny-weight
		24	
		463333	
		231668	
		<hr/>	
		Fac. 2780013	Grains

Multiply by 12, by 20,  
 and by 24, taking in the  
 Figures standing in the se-  
 veral Denominations; ac-  
 cording to the Direction  
 given in the seventh Rule  
 of this Chapter, and you  
 will find the Product to be  
 2780013 Grains, which is  
 the Number required, to  
 Answer to the Question.  
 See the whole Work, as in  
 the Margent.

*Quest.* 14. In 2780013 Grains, I demand how many  
 Pounds, Ounces, Penny-weights and Grains?

This is but the foregoing Question inverted, and is re-  
 solved by dividing by 24, by 20, and by 12, and the An-  
 swer is 482*lb.* 7 *oz.* 13 *pwt.* 21 *gr.*

2|0 12  
4) 2780013 (11583|3 (5791 (482 lb

24	10	48
38	15	99
24	14	96
140	18	31
120	18	24
200	3	Rem. 7 Ounces
192	2	

81 Rem. 13 Penny-weight

72

93

72

lb oz. pwt. gr.  
Facit 482 7 13 21

Remains 21 Grains.

Quest. 15. A Merchant sent to a Goldsmith 16 Ingots of Silver, each containing in Weight 2 lb 4 oz. and ordered it to be made into Bowls of 2 lb 10 oz. per Bowl, and Tankards of 1 lb 6 oz. per Piece, and Salts of 10 oz. 10 pwt. Salt, and Spoons of 1 oz. 18 pwt. per Spoon, and of each an equal Number; I desire to know how many of each sort he must make?

This Question is of the same Nature with the 11th and 12th Question foregoing, and may be answered after the same Method, viz. First, add the Weight of the several Vessels into which the Silver is to be made into one Sum, and reduce it to one Denomination, and they make 1248 Penny-weights; then reduce the Weight of the Ingot to the same Denomination, viz. Penny-weights, and it makes 560 Penny-weights, and multiply them by the Number of Ingots, viz. 16, and the Product will give you the Weight of the 16 Ingots, viz. 8960; then divide the Product by the Weight of the Vessels, viz. 1248, and the Quotient giveth you the Answer to the Question, viz. 7, and 224 pwt. remaineth over and above.

lb	oz.	lb	oz.
2	4	2	08
12		1	06
<hr/>		0	10
28		0	01
20		<hr/>	
<hr/>		Sum	5 02
560 Penny-weights		12	
16 Ingots		<hr/>	
<hr/>		62	
3360		20	
560		<hr/>	
<hr/>		1248	
1248)8960(7 Vessels of each			
8736			
<hr/>			
Rem. 224 Penny-weights			

*The Proof of the Work is as followeth, viz.*

			lb	oz.
7 {	Bowls	of 2 08 00 per Bowl, is	18	08
	Tankards	of 1 06 00 per Tank. is	10	06
	Salts	of 0 10 10 per Salt, is	06	01
	Spoons	of 0 01 18 per Spoon, is	01	01
	224 Penny-weight remaining		00	11
			<hr/>	
			37	04
			<hr/>	

So that you see the Sum of the Weight of each Vessel together with the Remainder, is 37lb 4oz. which is equal to the Weight of the 16 Ingots delivered; for if 37 4 oz. be reduced to Penny-weights, it makes 8960.

### *Réduction of Averdupois-weight.*

In reducing Averdupois-weights, the Learner must have Recourse to the Table of Averdupois-weight, delivered in the second Chapter.

P.  
2.  
8  
6  
0  
1  
2  
3  
6  
1  
4  
0  
4  
Veffe  
equ  
37  
0.  
t ha  
red  
Qu

Chap. 8.

Reduction.

73

Quest. 16. In 47 C. 1 gr. 20 lb  
how many Ounces? Multiply by 4,  
by 28, and 16, and the last Pro-  
duct will be the Answer, viz.  
84992 Ounces. See the Margent.

C. gr. lb

47 1 20

4

189 Quarters

28

1512

380

5312

16

31872

5312

Facit 84992 Ounces

Quest. 17. In 84992 Ounces, I demand how many  
C. grs. lb and oz.

This is the foregoing Question inverted, and will be  
resolved, if you divide by 16, by 28, and by 4, and the  
Answer is 47 C. 1 gr. 20 lb equal to the given Number in  
the foregoing Question.

16) 84992 (28) 4) C. gr. lb oz.  
(5312 (189 (47 1 20 00

80

28

16

49

251

29

48

224

28

19

272

(1) gr.

16

252

32

(20) lb

32

(0)

D

Reduction



## Reduction of Liquid Measure.

*Quest. 18.* In 45 Tuns of Wine, how many Gallons? Multiply by 4, and by 63, the Product is 11340 Gallons for the Answer.

$$\begin{array}{r}
 45 \\
 \times 4 \\
 \hline
 180 \\
 \times 63 \\
 \hline
 540 \\
 1080 \\
 \hline
 \end{array}$$

*Facit* 11340 Gallons

*Quest. 19.* In 34 Rundlets of Wine, each containing 18 Gallons, I demand how many Hogheads?

First, I find how many Gallons are in the 34 Rundlets, which you may do if you multiply 34 by 18, the Content of a Rundlet, and the Product is 612 Gallons, which you may reduce into Hogheads, if you divide them by 63, and the Quote will be 9 Hogheads and 45 Gallons. See the Work.

$$\begin{array}{r}
 34 \\
 \times 18 \\
 \hline
 272 \\
 34 \\
 \hline
 612
 \end{array}$$

63) 612 (9 hhds.

*Facit* 9 hhds, 45 Gal

Rem. 45 Gallons

*Quest. 20.* In 12 Tun, how many Rundlets of 14 Gallons per Rundlet.

Reduce your Tuns into Gallons, and divide them by 14, the Gallons in a Rundlet, and the Quotient 216, is your Answer. See the Work following.

$$\begin{array}{r}
 12 \\
 \times 4 \\
 \hline
 48 \\
 \times 63 \\
 \hline
 144 \\
 288 \\
 \hline
 14) 3024 (216 Rundlets
 \end{array}$$

$$\begin{array}{r}
 28 \\
 \times 22 \\
 \hline
 14 \\
 84 \\
 84 \\
 \hline
 \end{array}$$

(6) *Facit* 216 Rundlets.

## Reduction of Long Measure.

*Quest. 21.* I demand how many Furlongs, Poles, Inches and Barly-corns will reach from *London to York*, it being accounted 151 Miles?

151	Miles
8	Furlongs in a Mile
1208	Furlongs
40	Poles in a Furlong
48320	Poles
11	Half-yards in a Pole
48320	
48320	
531520	Half-yards
18	Inches in half a Yard
4252160	
531520	
9567360	Inches
3	Barly-corns in one Inch

*Facit* 28702080 Barly-corns in 151 *English* Miles.

*Quest. 22.* The Circumference of the Earth (as all other Circles are) is divided into 360 Degrees, and each Degree into 60 Minutes, which (upon the Superficies of the Earth) are equal to 60 Miles; now I demand how many Miles, Furlongs, Perches, Yards, Feet and Barly-corns will reach round the Globe of the Earth?

360	Degrees
60	Minutes or Miles in a Degree
21600	Miles about the Earth
8	Furlongs in a Mile
172800	Furlongs about the Earth
40	Perches in a Furlong
6912000	Poles or Perches about the Earth
11	Half-yards in a Perch
6912000	
6912000	
276032000	Half-yards upon the Earth
38016000	Yards, viz. the Half yards divided by 2
3	
114048000	Feet about the Earth
12	Inches in a Foot
228096000	
114048000	
1368576000	Inches about the Earth
3	Barly-corns in an Inch
4105728000	Barly-corns

And so many will reach round the World, the whole being about 21600 Miles; so that if any Person were to go round, and go 15 Miles every Day, he would go the whole Circumference in 1440 Days, which is 3 Years, 11 Months, and 15 Days.

*Reduction of Time.*

*Quest. 23.* In 28 Years, 24 Weeks, 4 Days, 16 Hours, 30 Minutes, how many Minutes?

Years	Weeks	Days	Hours	Min.
28	24	4	16	30
52 Weeks in a Year.				
<hr/>				
60				
142				
<hr/>				
1480	Weeks			
7				
<hr/>				
10364	Days			
24				
<hr/>				
41462				
20729				
<hr/>				
248752	Hours			
60				
<hr/>				
3492550				

*Note,* That in resolving the last Question after the Method expressed, there is lost in every Year, 30 Hours: for the Year consisteth of 365 Days and 6 Hours, but by multiplying the Year by 52 Weeks, which is but 364 Days, you lose 1 Day and 6 Hours every Year; wherefore to find an exact Answer, bring the odd Weeks, Days and Hours into Hours, and then multiply the Years by the Number of Hours in the Year, viz. 8766, and to the Product add the Hours contained in the odd Time, and you have the exact Time in Hours, which bring into Minutes as before. See the last Question thus resolved;

Week: Days Hours

24 4 16

7

172

24

694

345

4144

Hours add in  
(Mult.

Days Hours

365 6

24

1468

730

8766

Hours in a Year

28

8766

172

172

197

228

249592 Hours, add 30 min. in Mult.

66

14975550 Minutes in 28 Years, 4144 Hours and 30 Minutes.

So you see that according to the Method first used to resolve this Question, the Hours contained in the given Time are 248752; but according to the last, best, or truest Method, they are right, which exceeds the former by 340 Hours.

But for most Occasions it will be sufficient to multiply the given Years by 365, and to the Product add the Days in the odd Time, if there be any, and then there will be only a Loss of 6 Hours in every Year, which may be supplied by taking a fourth Part of the given Years, and adding it to the contained Days, and you have your Desire.

Quest. 24. In 458657340 Minutes, how many Years? Facit 834 Years, 4 Days, 19 Hours.

8766 Years Days Hours

610)438657340(7310959834

4 19

42

70128

18

29815

18

26298

6

35179

6

35064

57

24) 115 (4 Days.

54

96

35

Rcm. (19) Hours.

38

54

54

(0)

Quest. 25. I desire to know how many Hours and Minutes it is since the Birth of our Saviour Jesus Christ, being accounted 1766 Years.

This

This Question is of the same Nature with the 24th foregoing, and after the same Manner is resolved, viz. multiply the given Number of Years by 8766, the Product is 15393096 Hours, and that by 60, and the Product is 923585760 Minutes. See the Work.

$$\begin{array}{r}
 1756 \text{ Years} \\
 8766 \text{ Hours in a Year} \\
 \hline
 10536 \\
 10536 \\
 12292 \\
 14048 \\
 \hline
 15393096 \text{ Hours in 1756 Years} \\
 \hline
 60 \\
 \hline
 923585760 \text{ Minutes in 1756 Years}
 \end{array}$$

Note, That as Multiplication and Division do interchangeably prove each other, so Reduction descending and ascending prove each other by inverting the Question, as the 13th and 14th, and likewise the 16th and 17th Questions foregoing, by Inversion, do interchangeably prove each other. The like may be performed for the proof of any Question in Reduction whatsoever,

## CHAP. IX.

Of Comparative Arithmetick, viz. the Relation of Numbers one to another.

1. Comparative Arithmetick is that which is wrought by Numbers, as they are considered to have Relation one to another, and this consists either in Quantity or in Quality. *Vide Boëtus's Arith. lib. 1. cap. 21.*
2. Relation of Numbers in Quantity, is the Reference or Respect that the Numbers themselves have to one another, where the Terms or Numbers propounded are always two, the first called the Antecedent, and the other the Consequent. See Wing. *Arithm.*
3. The Relation of Numbers in Quantity consists in the Differences, or in the Rate or Reason that is found betwixt the Terms propounded, the Difference of two Numbers being the Remainder found by Subtraction (according to *Alsted*) but the Rate or Reason betwixt two Numbers is the Quotient of the Antecedent divided by the Consequent; for 21 and 7 being given, the Difference betwixt them will be found to be 14, but the Rate or Reason that is betwixt 21 and 7 will be found to be triple Reason, for 21 divided by 7 quotes 3, the Reason or Rate.
4. The



4. The Relation of Numbers in Quality (otherwise called Proportion) is the Reference or Respect that the Reason of Numbers have one unto another; therefore the Terms given ought to be more than two. Now this Proportion or Reason between Numbers relating one to another, is either Arithmetical or Geometrical.

5. Arithmetical Proportion is, when diverse Numbers differ one from another by equal Reason, that is, have equal Differences, (by some called Progression.)

So this Rank of Numbers, 3, 5, 7, 9, 11, 13, 15, 17, differ by equal Reason, viz. by 2, as you may prove.

6. In a Rank of Numbers that differ by Arithmetical Proportion, the Sum of the first and last Term being multiplied by half the Number of Terms, the Product is the total Sum of all the Terms.

Or, if you multiply the Number of Terms by the half Sum of the first and last Terms, the Product is the total Sum of all their Terms.

So in the former Progression given, 3 and 17 is 20, which multiplied by 4, viz. half the Number of Terms, the Product gives 80; the Sum of all the Terms: Or multiply 8 (the Number of Terms) by 10, half the Sum of the first and last Terms; the Product gives 80 as before.

So also 21, 18, 15, 12, 9, 6, 3, being given, the Sum of all the Terms will be found to be 84; for here the Number of Terms is 7, and the Sum of the first and last (viz. 21 and 3) is 24, half whereof (viz. 12) multiplied by 7, produceth 84, the Sum of the Terms sought.

7. Three Numbers that differ by Arithmetical Proportion, the Double of the Mean (or middle Number) is equal to the Sum of the Extremes.

So 9, 12 and 15 being given, the Double of the Mean 12 (viz. 24) is equal to the Sum of the two Extremes, 9 and 15.

8. Four Numbers that differ by Arithmetical Proportion (either continued or interrupted) the Sum of the two Means is equal to the Sum of the two Extremes.

So 9, 12, 18, 21, being given, the Sum of 12 and 18 will be equal to the Sum of 9 and 21, viz. 30: Also, 6, 8, 14, 16, being given, the Sum of 8 and 14 is equal to the Sum of 6 and 16, viz. 22, &c. See Wingate's *Arith.* v. 35.

9. Geometrical Proportion (by some called Geometrical Progression) is when diverse Numbers differ, according to like Reason.

So 1, 2, 4, 8, 16, 32, 64, &c. differ by double Reason, and 3, 9, 27, 81, 243, 729, differ by triple Reason; 4, 16, 64, 256, &c. differ by quadruple Reason, &c.

10. In any Numbers that increase by Geometrical Proportion, if you multiply the last Term by the Quotient of any one of the Terms divided by another of the Terms which being less is next unto it, and having deducted or subtracted the first Term out of that Product, divide the Remainder by a Number that is an Unit less than the said Quotient, the last Quote will be the Sum of all the Terms.

So 1, 2, 4, 8, 16, 32, 64, being given, first I take one of the Terms, viz. 8, and divide it by the Term which is less, and next to it (viz. by 4) and the Quotient is 2, by which I multiply the last Term 64, and the Product is 128, from whence I subtract the first Term (viz. 1) the Remainder is 127, which divided by the Quotient 2 made less by 1, viz. 1, the Quote is 127, for the Sum of all the given Terms, as by the Work in the Margent.

So if 4, 16, 64, 256, 1024, were given the Sum of all the Terms will be found to be 1364. For first I divide 64, one of the Terms, by the next lesser Term, and the Quotient is 4, by which I multiply the last Term 1024, and it produceth 4096; from whence I subtract the first Term 4, and the Remainder is 4092, which I divide by the Quote less by 1, viz. 3, and the Quote is 1364, for the total Sum of all the Terms, as per Margent.

11. Three Geometrical Proportions given, the Square of the Mean is equal to the Rectangle, or Product of the Extremes.

So 8, 16, 32, being given, the Square of the Mean, viz. 16, is 256, which is equal to the Product of the Extremes 8 and 32, for 8 times 32 is equal to 256.

12. Of four Geometrical proportionable Numbers given, the Product of the two Means is equal to the Product of the two Extremes.

So 8, 16, 32, 64, being given, I say, that the Product of the two Means, viz. 16 times 32, which is 512, is equal to 8 times 64, the Product of the Extremes.

Also if 3, 9, 21, 63 were given, which are interrupted, I say, 9 times 21 is equal to 3 times 63, which is equal to 189.

From hence ariseth that precious Gem in Arithmetick, which for the Excellency thereof is called the *Golden Rule*, or *Rule of Three*.

## C H A P. X.

*The Single Rule of Three Direct.*

**T**HE Rule of Three (not undeservedly called the Golden Rule) is that by which we find out a fourth Number in Proportion unto three given Numbers, so as this fourth Number that is sought may bear the same Rate, Reason and Proportion to the third (given) Number as the second doth to the first; from whence it is also called the *Rule of Proportion*.

2. Four Numbers are said to be proportional when the first containeth, or is contained by the second, as often as the third containeth, or is contained by the fourth. *Vide Wingate's Arith chap. 8. sect. 4.*

So these Numbers are said to be Proportionals, viz. 3, 6, 9, 18, for as often as the first Number is contained in the second, so often is the third contained in the fourth, viz. twice: Also 9, 3, 18, 6, are said to be Proportionals, for as often as the first Number containeth the second, so often the third Number containeth the fourth, viz. 3 times.

3. The Rule of Three is either simple or compound.

4. The simple (or single) Rule of Three consisteth of four Numbers, that is to say, it hath three Numbers given to find out a fourth; and this is either Direct or Inverse. *Vide Alsted. Math. lib. 2. c. 13.*

5. The single Rule of Three Direct, is when the Proportion of the first Term is to the second, as the third is to the fourth; or when it is required that the Number sought, viz. the fourth Number must have the same Proportion to the second, as the third hath to the first.

6. In the Rule of Three, the greatest Difficulty is to discover the Order of the 3 Terms of the Question propounded, viz. which is the first, second, and the third; which that you may understand, observe, that of the three given Numbers, two always are of one Kind; and the other is of the same Kind with the proportional Number that is sought; as in this Question, viz. If 4 Yards of Cloth cost 12 Shillings, what will 6 Yards cost at that Rate? Here the two Numbers of one Kind are 4 and 6, viz. they both signify so many Yards, and 12. is the same Kind with the Number sought, for the Price of 4 Yards is sought.

Again observe, That of the three given Numbers, those two that are of the same Kind, one of them must be the first, and the other the third, and that which is of the

same Kind with the Number sought, must be the second Number in the Rule of Three. And that you may know which of the said Numbers to make your first, and which your third, know this, that to one of those two Numbers there is always affixed a Demand, and that Number upon which the Demand lieth, must always be reckoned the third Number. As in the forementioned Question, the Demand is affixed to the Number 6; for it is demanded, what 6 Yards will cost, and therefore 6 must be the third Number, and 4 (which is of the same Denomination or Kind with it) must be the first, and consequently the Number 12 must be the second; and then the Numbers being placed in the forementioned Order, will stand as followeth, *viz.*

<i>Yards</i>	<i>s.</i>	<i>Yards</i>
4	12	6

7. The next Thing is, to find out the fourth Number in Proportion; which that you may do, multiply the second Number by the third, and divide the Product thereof by the first, or (which is all one) multiply the third Term (or Number) by the second, and divide the Product thereof by the first, and the Quotient thence arising is the 4th Number in a direct Proportion, and is the Number sought, or Answer to the Question, and is of the same Denomination that the second Number is of; as thus, let the same Question be again repeated, *viz.* If 4 Yards of Cloth cost 12 Shillings, what will 6 Yards cost?

Having placed my Numbers according the 6th Rule (of this Chapter) foregoing, I multiply the second Number 12, by the third Number 6, and the Product is 72, which Product I divide by the first Number 4, and the Quotient thence arising is 18, which is the fourth Proportional or Number sought, *viz.* 18 Shillings, (because the second Number is Shillings) which is the Price of 6 Yards, as was required by the Question. See the Work following.

<i>Yards</i>	<i>s.</i>	<i>Yards</i>	<i>s.</i>
4	12	6	18
<hr/>			
4) 72 (18 Shillings			
<hr/>			
4			
<hr/>			
32			
<hr/>			
32			
<hr/>			
(0)			



*Quest. 2.* Another Question may be this, *viz.* If 7 C. of Pepper cost 21*l.* how much will 16 C. cost at that Rate?

To resolve which Question I consider that (according to the 6th Rule of this Chapter) the Terms or Numbers ought to be placed thus, *viz.* the Demand lying upon 16C. it must be the third Number, and that of the same Kind with it must be the first, *viz.* 7C.; and 21*l.* (being of the same Kind with the Number sought) must be the second Number in this Question; then I proceed according to this 7th Rule, and multiply the second Number by the third, *viz.* 21 by 16, and the Product is 336, which I divide by the first Number 7, and the Quotient is 48*l.* which is the Value of 16C. of Pepper at the Rate of 21*l.* for 7C. See the Work following.

$$\begin{array}{r}
 \text{C.} \quad \text{l.} \quad \text{C.} \\
 7 \quad 21 \quad 16 \\
 \hline
 126 \\
 21 \\
 \hline
 336 \\
 7 \overline{) 336} \quad 48 \text{ } 400 \\
 \hline
 28 \\
 \hline
 56 \\
 \hline
 56 \text{ Facit } 48 \text{ } l. \\
 \hline
 (0)
 \end{array}$$

8. If when you have divided the Product of the second and third Numbers by the first, any Thing remain after Division is ended, such Remainder may be multiplied by the Parts of the next inferior Denomination, that are equal to an Unit (or Integer) of the second Number in the Question, and the Product thereof divide by the first Number in the Question, and the Quotient is of the same Denomination with the Parts by which you multiplied the Remainder, and is Part of the fourth Number which is sought. And furthermore, if any Thing remain after this last Division is ended, multiply it by the Parts of the next inferior Denomination, equal to an Unit of the last Quotient, and divide the Product by the same Divisor, (*viz.* the first Number in the Question, and the Quote is still of the same Denomination with your Multiplier; follow this Method until you have reduced your Remainder into the lowest Denomination, &c. An Example or two will make this Rule very plain, which may be the following.

*Quest.*



*Quest. 3.* If 13 Yards of Velvet, &c. cost 21 $\frac{1}{2}$  what will 27 Yards of the same cost at that Rate?

Having ordered and wrought my Numbers according to the 6th and 7th Rules of this Chapter, I find the Quotient to be 43 $\frac{1}{2}$  and there is a Remainder of 8, so that I conclude the Price of 27 Yards to be more than 43 $\frac{1}{2}$  and to the Intent that I may know how much more, I work according to the foregoing Rule, *viz.* I multiply the said Remainder 8 by 20s. (because the second Number in the Question was Pounds) and the Product is 160, which divided by the first Number, *viz.* 13, it quotes 12, which are 12 Shillings, and there is yet a Remainder of 4, which I multiply by 12 Pence, (because the last Quotient was Shillings) and the Product is 48, which I divide by 13 (the first Number) and the Quotient is 3 $\frac{1}{2}$  and yet there remaineth 9, which I multiply by 4 Farthings, and the Product is 36, which divided by 13 again, it quotes 2 Farthings, and there is yet a Remainder of 10, which (because it cometh not to the Value of a Farthing) may be neglected, or rather set after the 2 Farthings over the Divisor with a Line between them, and then (by the 21st and 22d Definitions of the first Chapter of this Book) it will be  $\frac{1}{3}$  of a Farthing; so that I conclude, that if 13 Yards of Velvet cost 21  $\frac{1}{2}$  27 Yards of the same will cost 43  $\frac{1}{2}$  12 s. 3 d. 2 $\frac{1}{3}$  grs. which Fraction is 10 Thirtieths of a Farthing. See the Operation as followeth.

$\begin{array}{r} \text{yds.} \quad \text{l.} \quad \text{yds.} \\ \text{If } 13 \quad 21 \quad 27 \\ \quad \quad 27 \\ \hline \quad \quad 147 \\ \quad \quad 42 \\ \hline 13) 567 \quad (43\frac{1}{2} \end{array}$

$$\begin{array}{r} 52 \\ \hline 47 \\ 39 \\ \hline \end{array}$$

Remain (8)  
Multiply 20

$$\begin{array}{r} 13 \overline{) 160} \quad (12) \\ \underline{156} \phantom{0} \\ 40 \phantom{0} \\ \underline{39} \phantom{0} \\ 10 \phantom{0} \\ \underline{9} \phantom{0} \\ 1 \end{array}$$

Remains (4)  
Multiply 12

---

13) 48 (3 d.  
39

Remain (9)  
Multiply 4

	—	l.	s.	d.	grs.
Remain	10 Facit	43	12	3	2 $\frac{0}{4}$

*Quest. 4.* Another Example may be this following.  
viz. If 14 Pounds of Tobacco cost 27 s. what will 478  
Pounds cost at that Rate?

## Work

Work according to the last Rule, and you will find it to amount to 92 *ls.* 10*d.* 1  $\frac{2}{4}$  *qrs.* and by the 5th Rule of the 8th Chapter 92 *ls.* may be reduced to 46*l.* 1*s.* so that then the whole Worth or Value of the 478*l.* will be 46*l.* 1*s.* 10*d.* 1  $\frac{2}{4}$  *qrs.* The Work followeth.

l.    s.    l.  
If 14 : 27 :: 478

27

3346

956

2|0

14) 12906 (92 | 1 (46*l.*

126    8

30    12

28    12

26 (1)

14

Remains (12)

Multiply 12

24

12

14) 144 (10*d.*

14

Remains (4)

Multiply 4

14) 16 (1  $\frac{2}{4}$

Remains (2)

*Facit.* 46*l.* 1*s.* 10*d.* 1  $\frac{2}{4}$

In the Rule of Three it many times happens, that al-  
 though the first and third Numbers be of one Kind, as both  
 Money, Weight, Measure, &c. yet they may not be of one  
 Denomination, or perhaps they may both consist of many  
 Denominations; in which Case you are to reduce both  
 Numbers to one Denomination, and likewise your second  
 Number (if it consisteth at any time of diverse Denomi-  
 nations) must be reduced to the least Name mentioned, or  
 lower if you please; which being done, multiply the se-  
 cond and third together, and divide by the first, as is di-  
 rected in the 7th Rule of this Chapter.

And note, that always the Answer to the Question is in  
 the same Denomination that your second Number is of,  
 as is reduced to, as was hinted before.

Quest. 5. If 15 Ounces of Silver be worth 3*l*. 15*s*. what  
 will 86 Ounces worth at that Rate?

In this Question the Numbers being ordered according  
 to the 6th Rule of this Chapter, the first and third Num-  
 bers are Ounces, and the second Number is of diverse  
 Denominations, viz. 3*l*. 15*s*. which must be reduced to  
 Shillings, and the Shillings multiplied by the third Num-  
 ber, and the Product divided by the first, gives you the  
 Answer in Shillings, viz. 430 Shillings, which are reduced  
 to 21*l*. 10*s*.

$$\begin{array}{r}
 \text{oz.} \quad \text{l.} \quad \text{s.} \quad \text{oz.} \\
 \text{If } 15 : 3 \quad 15 :: 86 \\
 \hline
 20 \\
 75 \\
 86 \\
 \hline
 450 \\
 600 \\
 \hline
 20 \text{ } \text{l.} \quad \text{s.} \\
 15) 6450 (430 (21 \quad 10 \\
 \hline
 60 \quad 4 \\
 45 \quad 3 \\
 45 \quad 2 \\
 \hline
 (0) \quad (10) \text{s.}
 \end{array}$$

In resolving the last Question, the Work would have  
 been the same if you had reduced your second Number  
 into Pence, for then the Answer would have been 5160  
 Pence, equal to 21*l*. 10*s*. or if you had reduced the second  
 Number into Farthings, the Quotient or Answer would  
 have been 20640 Farthings, equal to the same, as you may  
 prove at your Leisure.

Quest.

*Quest. 6.* If 8 lb of Pepper cost 4s. 8d. what will 7 C. 3 qrs. 14 lb cost?

In this Question the first Number is 8 lb and the third is 7 C. 3 qrs. 14 lb which must be reduced to the same Denomination with the first, viz into Pounds and the second Number must be reduced into Pence; then multiply and divide according to the 7th Rule foregoing, and you will find the Answer to be 6174 Pence, which is reduced into 25l. 14s. 6d.

lb. s. d. C. qrs. lb  
If 8 cost 4 8 what will 7 3 14 cost?

12

4

56

31

28

252

63

882

56 second Number

5292

4410

8) 49392 (6174 2|0) 7. s. d.  
514 (25 14 6

48

60

4

13

17

11

8

12

10

59

54

(14)s.

56

48

32

(6)d.

32

l.

s.

d.

(6) Facit 25 14 6



Quest. 7. If 3C. 1qr. 14lb of Raisins cost 9l. 9s. what will 6C. 3qrs. 20lb of the same cost?

Here the first and third Numbers each consist of diverse Denominations, but must be brought both into one Denomination, &c. as you see in the Operation that followeth.

If 3 1 14 cost 9l. 9s. what will 6 3 20 cost?

4	20	4
13	189	27
28		28
108		216
27		56

378 Pounds

776 Pounds

189 second Number

6984  
6208  
776

378) 146664 (388 19 8

1134 2  
3326 18  
3024 18

3024 (8)

3024

Facit 19 8

(0)

Quest. 8 If in 4 Weeks I spend 13s. 4d. how long will 3l. 6s. last me at that Rate?

Answer 2238 Days, equal to 6 Years, 48 Days. See the Work.

<i>s.</i>	<i>d.</i>	<i>W.</i>	<i>l.</i>	<i>s.</i>
If 13	4	require 4	what will 53	6 require ?
<u>12</u>		<u>7</u>	<u>20</u>	
30		28 Days	1066	
<u>13</u>			<u>12</u>	
160			2132	
			<u>1066</u>	
			12792 Pence	
			28 second Number	
			<u>102336</u>	
			25584	
			<u>365</u>	
		16 0	35817 6	(2238 (6 Years
			...	2190
		<u>32</u>		
			Rem. (48) Days	
		<u>38</u>		
		<u>32</u>		
			61	Years
			48	Facit 6 48 $\frac{1}{2}$ 6
			<u>137</u>	
			128	

Remains (96)

*Quest. 9.* Suppose the yearly Rent of a House, a year Pension, or Wages, be 73*l.* I desire to know how much it is *per Day* ?

Here you are to bring the Year into Days, and say, 365 Days require 73*l.* what will one Day require ?

Now when you come to multiply 73 by 1, the Product is the same, for 1 neither multiplieth nor divideth ; and 73 cannot be divided by 365, because the Divisor is bigger than the Dividend ; wherefore bring the 73*l.* into Shillings, and they make 1460, which divide by the first Number, 365, and the Quote is 4 Shillings for the Answer ; as you see in the Work.

Days 1. Day 1  
If 365 73 1

20  
365) 1460 (4 s.

1460

Facit 4 s. per Day.

(0)

Quest. 10. A Merchant bought 14 Pieces of Broad-cloth, each Piece containing 28 Yards; for which he gave after the Rate of 13s. 6d.  $\frac{1}{2}$  per Yard, now I desire to know how much he gave for the 84 Pieces at that Rate?

First find out how many Yards are in the 14 Pieces, which you will do if you multiply the 14 Pieces by 28 (the Number of Yards in a Piece) and it makes 392; then say, if 1 Yard cost 13s. 6d.  $\frac{1}{2}$  what will 392 Yards cost? Work as followeth, and the Answer you will find to be 127400 Half-pence, which reduced makes 265 l. 8s. 4d. for after you have multiplied your second and third Numbers together, the Product is 127400, which (according to the seventh Rule) should be divided by the first Number; but the first Number is 1, which neither multiplieth nor divideth, and therefore the Quotient, or fourth Number, is the same with the Product of the second and third, which is in Half-pence, because the second Number was so reduced. See the Work as followeth.

28  
14  
112  
28

392 Yards in the 14 Pieces.

Yd. s. d. Yds.  
If 1 cost 13 6  $\frac{1}{2}$ , what will 392 cost?

12  
32  
13  
162

02  
(0)

325  
1960  
784  
1176

the second Number.

20  
24) 127400 (5308 (265 l.

Half-pence 325

120  
74  
72  
200  
192

4  
13  
12  
10  
10

Facit 265 l. 8s. 4d.

Remains (8)

(8) Shillings.

Half-pence, or 4d.

Quest.

*Quest. 11.* A Draper bought 420 Yards of Broadcloth, and gave for it after the Rate of 14s. 10d.  $\frac{3}{4}$  per Ell *English*, now I demand how much he paid for the Whole after that Rate?

Bring your Ells into Quarters, and your given Yards into Quarters; the Ell is 5 Quarters, and in 420 Yards are 1680 Quarters; then say, if 5 Quarters cost 14s. 10d.  $\frac{3}{4}$  (or 715 Farthings) what will 1680 Quarters cost? *Facit* 250l. 5s. See the Operation.

Ells	Yards
420	420
<u>5</u>	<u>4</u>
1680	1680
<i>grs.</i> 14s. 10d. $\frac{3}{4}$	
If 5 : 14s. 10d. $\frac{3}{4}$ :: 1680	
<u>12</u>	<u>715</u>
28	8400
<u>15</u>	<u>1680</u>
178 d.	11760
<u>4</u>	<u>96</u>
	5) 1201200 (240240 (250l. 5s.)
715 <i>grs.</i>	

l.	s.	d.
250	5	0
<i>Facit</i> 250	5	0

*Quest. 12.* A Draper bought of a Merchant 50 Pieces of Kersey, each Piece containing 34 Ells *Flemish* (the Ell *Flemish* being three Quarters of a Yard) to pay after the Rate of 8s. 4d. per Ell *English*; I demand how much the 50 Pieces cost him at that Rate?

First find out how many Ells *Flemish* are in the 50 Pieces by multiplying 50 by 34, the Product is 1700, which bring into Quarters by 3, it makes 5100 Quarters; then proceed as in the last Question, and the Answer you will find to be 102000 Pence, or 425l. See the Operation as followeth.

grs.	s.	d.	grs.
If 5	: 8	4	: : 5100
12			100
<hr/>		<hr/>	
1004		5	310000 (102000 200
			150
		5	: : 1700 Ells Fl.
		10	
		10	
		<hr/>	
		5100	
		(0)	(2 0
		<hr/>	
		12) 102000 (850 0 (425/	

96	8
<hr/>	<hr/>

60	5
----	---

60	4
----	---

(0)	10
-----	----

	10
--	----

(0)	
-----	--

Facit 425/

Quest. 13. A Goldsmith bought a Wedge of Gold which weighed 14th 3oz. 8pwt. for the Sum of 514l. 4s. I demand what it stood him in per Ounce? Answer 60s. or 3l.

lb	oz.	pwt.	l.	s.	oz.
If 14	3	8	: : 514	4	: : 1
12				20	Shil.
<hr/>		<hr/>		<hr/>	

31
----

14
----

171 oz.
---------

20
----

3428 pwt.
-----------

10284
-------

20 pwt.
---------

2 0
-----

3428) 205680 (6 0 (3l.
------------------------

6
---

205680
--------

(0) Facit. 60s. or 3l.
------------------------

(0)
-----

Quest.



*Quest, 14.* A Grocer bought 4 Hogheads of Sugar, each weighing neat 6 C. 2 qrs. 14lb which cost him 2l. 8s. 6d. per C. I demand the Value of the 4 Hhds, at that Rate?

First find the Weight of the 4 Hhds, which you may do by reducing the Weight of one of them into Pounds, and multiply them by 4 (the Number of Hhds) and they make 2968lb: then say, if 1 C. or 112lb cost 2l. 8s. 6d. what will 2968lb cost? *Facit* 64l. 5s. 3d. as by the Operation.

		C.	qrs.	lb
		6	2	14
		4		
		26		
		28		
		212		
lb	l. s. d.	lb		
If 112 : 2 8 6 :: 2968		53		
20		582		
48		5936	742lb in 1 hhd.	
12		23744	4 hogheads	
102		14840	2968 lb in 4 hhds.	
48		112	12	210
582		1727376	15423	12815 (64l.
		112	12	12
		607	34	8
		560	24	8
		473	102	(5) Shillings
		448	96	
		257	63	
		224	60	
		336	(3) Pence	
		336		
		(0)	<i>Facit</i> 64l. 5s. 3d.	

Quest. 15. A Draper bought of a Merchant 8 Packs of cloth, each containing 4 Parcels, and each Parcel 10 Pieces, and each Piece 26 Yards, and gave after the Rate of 4*l*. for 6 Yards, now I desire to know how much he gave for the Whole? *Answer* 6656*l*.

First find out how many Yards there were in the 8 Packs, and by the following Work you will find there are 8320 Yards: then say, if 6 Yards cost 4*l*. 16*s*. what will 8320 Yards cost, &c.

8 Packs  
4  
32 Parcels  
10  
320  
26  
8320 Yards

1. s. Yds.  
6 : 4 16 :: 8320  
20 96  
96 49920  
74880

210  
6) 798720 (1331210 (6656*l*.

6 12  
19 13  
18 12  
18 11  
18 10  
7 12  
6 12  
12 (0)  
12  
(0)

6656*l*.

By this time the Learner is, as I suppose, well exercised in the Practick and Theorick of the Rule of Three Direct; at his Leisure he may look over the following Questions, whose Answers are given; but the Operation purposely omitted as a Touchstone for the Learner, thereby to try his Ability in what hath been deliver'd in the former Rules.

Quest. 16. If 24*l*. of Raisins cost 6*s*. 6*d*, what will 18 Fraills cost, each weighing neat 39*rs*. 18*lb*. *Answer* 24*l*. 17*s*. 3*d*.

Quest. 17. If an Ounce of Silver be worth 5 Shillings, what is the Price of 14 Ingots, each Ingot weighing 7*lb*. 10*pw*t. *Answer* 313*l*. 5*s*.

Quest.

*Quest. 18.* If a Piece of Cloth cost 10*l.* 16*s.* 8*d.* I demand how many Ells *English* there are in the same, when 1 Ell at that Rate is worth 8*s.* 4*d.*? *Answer* 26 Ells *English*.

*Quest. 19.* A Factor bought 84 Pieces of Stuffs, which cost him in all 537*l.* 12*s.* at 5*s.* 4*d.* per Yard, I demand how many Yards there were in all, and how many Ells *English* were contained in a Piece of the same? *Answer* 2016 Yards in all, and 19½ Ells *English* per Piece.

*Quest. 20.* A Draper bought 242 Yards of Broad-cloth which cost him in all 254*l.* 10*s.* for 86 Yards of which he gave after the Rate of 21*s.* 4*d.* per Yard. I demand how much he gave per Yard for the Remainder? *Answer* 20*s.* 10*d.* ⅔ per Yard.

*Quest. 21.* A Factor bought a certain Quantity of Serge and Shalloon, which together cost him 26*l.* 14*s.* 40*d.* The Quantity of Serge he bought was 48 Yards, at 4*s.* 4*d.* per Yard; and for every two Yards of Serge he had 5 Yards of Shalloon; I demand how many Yards of Shalloon he had, and how much the Shalloon cost him per Yard?

*Ans.* 120 Yards of Shalloon at 2*s.* 8*d.* ⅔ per Yard?

*Quest. 22.* An Oilman bought three Tuns of Oil, which cost him 15*l.* 14*s.* and so it chanced that it leaked out 10 Gallons; but he is minded to sell it again, so that he may be no Loser by it; I demand how he must sell it per Gallon? *Answer*, at 4*s.* 6*d.* ⅔ per Gallon.

*Quest. 23.* Bought 6 Packs of Cloth, each Pack containing 12 Cloths, which at 8*s.* 4*d.* per Ell *Flemish*, cost 108*l.* I demand how many Yards there were in each Cloth? *Answer*, 27 Yards in each Cloth?

*Quest. 24.* A Gentleman hath 536*l.* per Ann. and his Expences are, one Day with another 18*s.* 10*d.* 3*qrs.* I desire to know how much he layeth up at the Year's End? *Answer* 191*l.* 3*s.* 0*d.* 1*qr.*

*Quest. 25.* A Gentleman expendeth daily one Day with another 27*s.* 10*d.* ½, and at the Year's End layeth up 340*l.* I demand how much is his yearly Income? *Answer* 848*l.* 14*s.* 4*d.*

*Quest. 26.* If I sell 14 Yards for 10*l.* 10*s.* how many Ells *Flemish* shall I sell for 283*l.* 17*s.* 6*d.* at that Rate? *Answer* 504 ⅔ Ells *Flemish*.

*Quest. 27.* If 100*l.* in 12 Months, gain 6*l.* Interest, how much will 75*l.* gain in the same Time, and at the same Rate? *Answer* 4*l.* 10*s.*

*Quest. 28.* If 100*l.* in 12 Months, gain 6*l.* Interest; how much will it gain in 7 Months at that Rate? *Ans.* 3*l.* 10*s.*

*Quest. 29.* A certain Usurer put out 75*l.* for 12 Months, and received Principal and Interest 81*l.* I demand at what Rate *per Cent.* he received Interest? *Answer* 8*l.* *per Cent.*

*Quest. 30.* A Grocer bought 2 Chests of Sugar, the one weigh'd neat 18*C.* 3*qrs.* 14*lb.* at 2*l.* 6*s.* 8*d.* *per C.* the other weigh'd neat 18*C.* 1*qr.* 21*lb.* at 4*d.*  $\frac{1}{2}$  *per lb.* which he mingled together; now I desire to know how much a *C.* wt. of this Mixture is worth? *Answer* 2*l.* 4*s.* 4*d.*  $1\frac{3}{4}$  *qrs.*

*Quest. 31.* Two Men, *viz.* A and B departed both from one Place, the one goes East, and the other West; one travelleth 4 Miles a Day, and the other 5 Miles a Day, how far are they distant, the 9th Day after their Departure? *Answer* 81 Miles.

*Quest. 32.* A, flying every Day 40 Miles; is pursued the fourth Day after by B. posting 50 Miles a Day? Now the Question is, in how many Days, and after how many Miles Travel will A be overtaken?

*Ans.* B overtakes him in 12 Days, when they have travelled 600 Miles. See *Moré's Arithm.* cap. 8. qu 7.

11. The general Effect of the Rule of Three Direct, is contained in the Definition of the same, that is, to find a fourth Number in Proportion; consisting of two equal Reasons; as hath been fully shewn in all the foregoing Examples.

The second Effect is, by the Price or Value of one Thing, to find the Price and Value of many Things of like Kind.

The third Effect is, by the Price or Value of many Things, to find the Price of one; or by the Price of many Things, (the said Price being one) to find the Price of many Things of like Kind.

The fourth Effect is, by the Price or Value of many Things, to find the Price or Value of many Things of like Kind.

The fifth Effect is, thereby to reduce any Number of Moneys, Weights, or Measures, the one Sort into the other, as in the Rules of Reduction contained in the eighth Chapter foregoing. Examples of its various Effects have been already answered.

12. The Rule of Three Direct is thus proved, *viz.* multiply the first Number by the fourth, and note the Product; then multiply the second Number by the third, and if this Product is equal to the Product of the first and fourth, then the Work is rightly performed, otherwise it is erroneous.

*The Proof of the Rule of Three Direct.*

So the first Question of this Chapter, (whose Answer or fourth Number we found to be 18s.) is thus proved, *viz.* the first Number is 4, which multiplied by 18 (the fourth) produceth 72, and the second and third Numbers are 12 and 16, which multiplied together produce 72, equal to the Product of the first and fourth, and therefore I conclude the Work to be rightly performed.

Always observing, that if any Thing remain after you have divided the Product of the second and third Numbers by the first, such Remainder in proving the same must be added to the Product of the first and fourth Numbers, whose Sum will be equal to the product of the second and third, the second Number being of the same Denomination with the fourth, and the first of the same Denomination with the third.

So the fourth Question of this Chapter being again repeated, *viz.* if 14 lb of Tobacco cost 27s. what will 478 lb cost at that Rate? The Answer (or fourth Number) was 46l. 1s. 10d. 1qr.  $\frac{3}{4}$ , which is thus proved, *viz.* bring the fourth Number into Farthings, and it makes 44249, which multiplied by the first Number 14, produceth 619488 (the second which remaineth being added thereto); then, because I reduce my fourth Number into Farthings, I reduce my second (*viz.* 27s.) into Farthings, and they are 1296, which multiplied by the third Number 478, their Product is 619488, equal to the product of the first and fourth Numbers, wherefore I conclude the Operation to be true. This is an infallible Way to prove the Rule of Three Direct, and it is deduced from the 12th Section of the 9th Chapter of this Book.

And thus much for this inestimable Rule of Three Direct, the Demonstration of which may be seen in *Kersey's Appendix to Wingate's Arithmetick*, and in the 6th Chapter of *Oughtred's Clavis Mathematicæ*.

## C H A P. XI.

### *The Single Rule of Three Inverse.*

1. **T**HE Golden Rule, or Rule of Three Inverse, is when there are three Numbers given, to find a fourth in such proportion to the three given Numbers, so as the fourth proceeds from the second, according to the same Rate, Reason or proportion, that the first proceeds from the third, or the proportion is.



As the third Number is in proportion to the second, so is the first to the fourth. See *Alfred. Math.* l. 2. c. 14.

So if the three given Numbers were 8, 12 and 16, and it were required to find a fourth Number in an inverted proportion to these, I say, that as 16 (the third Number) is the Double of the first Term or Number (8) so must 12, the second Number, be the Double of the fourth; so will you find the fourth Term or Number to be 6. And (as in the Rule of Three Direct) you multiply the second and third together, and divide their Product for a fourth proportional Number.

2. In the Rule of Three Inverse, you must multiply the second Term by the first, or first Term by the second, and divide the Product thereof by the third Term, so the Quotient will give you the fourth Term sought in an inverted Proportion. The same Order being observed in this Rule as in the Rule of Three Direct, for placing and disposing of the given Numbers, and after your Numbers are placed in Order, that you may know whether your Question be to be resolved by the Rule Direct or Inverse, observe the general Rule following.

3. When your Question is stated, and your Numbers orderly disposed, consider in the first Place whether the fourth Term or Number sought ought to be more or less than the second Term, which you may easily do; and if it is required to be more or greater than the second Term, then the lesser Extreme must be your Divisor; but if it requires less, then the highest Extreme must be your Divisor; in this Case the first and third Numbers are called Extremes (in respect of the second) and having found out your Divisor, you may know whether your Question belong to the Rule Direct or Inverse; for if the third Term be your Divisor, then it is Inverse, but if the first Term be your Divisor, then it is a Direct Rule: As in the following Questions.

*Quest. 1.* If 8 Labourers can do a certain piece of Work in 12 Days, in how many Days will 16 Labourers do the same? *Answer,* in 6 Days.

Having placed the Numbers according to the 6th Rule of the 10th Chapter, I consider, that if 8 Men can finish the Work in 12 Days, 16 Men will do it in lesser (or fewer Days) than 12, therefore the bigger Extreme must be the Divisor, which is 16, and therefore it is the Rule of Three Inverse; wherefore I multiply the first and second Numbers together, viz. 8 by

Lab.	Days	Lab.
8	12	16
	8	
	<hr style="width: 50%; margin: 0 auto;"/>	
	16) 96	(6 Days
	96	
	<hr style="width: 50%; margin: 0 auto;"/>	
	(0)	
	Facit 6 Days.	

12 the Product is 96, which divided by 16 the third Term quotes 6 Days for the Answer: and in so many Days will 16 Labourers perform a piece of Work, when 8 Men can do it in 12 Days.

*Quest. 2.* If when the Measure, *viz.* (a Peck) of Wheat cost 2*s.* the Penny Loaf weighed (according to the Standard, Statute or Law of England) 8 Ounces, I demand how much it will weigh when the Peck is worth 1*s.* 6*d.* according to the same Rate or proportion? *A sw.* 10 oz. 13 pwt. 8 gr.

Having placed and reduced the given Numbers according to the 6th and 9th Rules of the 10th Chapter, I consider that at 1*s.* 6*d.* per Peck, the Penny Loaf will weigh more than at 2*s.* per Peck; for as the Price decreaseth, the Weight increaseth; and as the Price increaseth, so the Weight diminishes; wherefore, because the first Term requires more than the second, the lesser Extreme must be the Divisor, *viz.* 1*s.* 6*d.* or 18*d.* and having finished the Work, I find the Answer to be 10 oz. 13 pwt. 8 gr. and so much will the Penny Loaf weigh when the Peck of Wheat is worth 1*s.* 6*d.* according to the given Rate of 8 Ounces when the Peck is worth 2*s.* The Work is plain in the following Operation

$$\begin{array}{r}
 \begin{array}{ccc} d. & oz. & dr. \\ \text{If } 24 & 8 & 18 \\ & 8 & \end{array} \\
 \hline
 \text{-----} oz. pwt. gr. \\
 18) 192 (10 \quad 13 \quad 8 \text{ Ans. } \\
 \quad 18 \\
 \hline
 \quad 12 \\
 \quad 20 \\
 \hline
 \text{-----} pwt. \\
 240 (13 \\
 \quad 18 \\
 \hline
 \quad 60 \\
 \quad 34 \\
 \hline
 \quad 6) \\
 \quad 24 \\
 \hline
 144 (8 gr. \\
 144 \\
 \hline
 (0)
 \end{array}$$

**Quest. 3.** How many Pieces of Money or Merchandize, at 20 s. per Piece, are to be given or received for 240 Pieces, the Value of every Piece being 12 shillings? *Ans.* 14 Pieces. For 12 s. required 240 Pieces, then 20 s. will require less; therefore the bigger Extreme must be the Divisor, which is the third Number, &c. See the Work as in the Margent.

s.	pcs.	s.
If 12	240	20
	<u>12</u>	
	480	
	<u>240</u>	
240)	2880	(144 pcs. at
	..	20s. per pc.
	<u>2</u>	
	8	
	<u>8</u>	
	8	
	<u>8</u>	
	(0)	

**Quest. 4.** How many Yards of 3 Quarters broad are required to double or be equal in Measure to 30 Yards that are 5 Quarters broad? *Answer,* 50 Yards. For say, if 5 Quarters will require 30 Yards long, what Length will 3 Quarters broad require? Here I consider that 3 Quarters broad will require more Yards than 30; for the narrower the Cloth is, the more in Length will go to make equal Measure with a broader Piece.

grs.	long	grs.
5	30	3
	<u>5</u>	
3)	150	(50 Yards
	<u>15</u>	
	(0)	

**Quest. 5.** At the Request of a Friend I lent him 200l. for 12 Months, promising to do me the like Courtesy at my Necessity; but when I came to request it of him, he could let me have but 150l. now I desire to know how long I may keep this Money to make plenary Satisfaction for my former Kindness to my Friend? *Ans.* 16 Months. I say, if 200l. require 12 Months, what will 150l. require. 150l. will require more Time than 12 Months, therefore the lesser Extreme (*viz.* 150) must be the Divisor; multiply and divide, and you will find the fourth inverted Proportional to be 16, and so many Months I ought to keep the 150l. for Satisfaction.

**Quest. 6.** If for 24 s. I have 1200lb Weight carried 36 Miles, how many Miles shall 1800lb be carried for the same Money? *Ans.* 24 Miles.

**Quest. 7.** If for 24 s. I have 1200lb Weight carried 36 Miles, how many lb Weight shall I have carried 24 Miles for the same Money? *Ans.* 1800 lb Weight.

*Quest.*

*Quest. 8.* If 100 Workmen in 12 Days finish a Piece of Work or Service, how many Workmen are sufficient to do the same in 3 Days? *Answer*, 400 Workmen.

*Quest. 9.* A Colonel is besieged in a Town, in which are 1000 Soldiers, with Provision of Victuals only for 3 Months the Question is, how many of his Soldiers must he dismiss that his Victuals may last the remaining Soldiers 6 Months? *Answer* 500 he must keep, and dismiss as many.

*Quest. 10.* If 20*l.* worth of Wine is sufficient for the Ordinary of 100 Men, when the Tun is sold for 30*l.* how many Men will the same 20*l.* worth suffice, when the Tun is worth 24*l.*? *Answer* 125 Men.

*Quest. 11.* How much Plush is sufficient for the Cloak which hath in it 4 Yards of 7 Quarters wide, when the Plush is but 3 Quarters wide? *Answer*.  $9\frac{1}{3}$  Yards of Plush.

*Quest. 12.* How many Yards of Canvas, that is Ell wide, will be sufficient to line 20 Yards of Say, that is 3 Quarters wide? *Answer* 12 Yards.

*Quest. 13.* How many Yards of Mattin that is 2 Foot wide, will cover a Floor that is 24 Foot long and 20 Foot broad? *Answer* 80 Yards.

*Quest. 14.* A Regiment of Soldiers consisteth of 1000, are to have new Coats, and each Coat to contain two Yards two Quarters of Cloth that is 5 Quarters wide, and they are to be lined with Shalloon that is 3 Quarters wide, I demand how many Yards of Shalloon will line them? *Answer* 16666 $\frac{2}{3}$  Quarters, or 4166 $\frac{2}{3}$  Yards.

*Quest. 15.* A Messenger makes a Journey in 24 Days when the Days are 12 Hours long; I desire to know in how many days he will go the same, when the Day is 16 Hours long? *Answer* in 18 Days.

*Quest. 16.* I borrowed of my Friend 6*l.* for 8 Months and he hath Occasion another Time to borrow of me for 12 Months I desire to know how much I must lend to make good his former Kindness to me? *Answer* 42*l.* 13*s.* 4*d.*

4. The general Effect of the Rule of Three Inverse, is contained in the Definition of the same, that is, to find a fourth Term in a reciprocal Proportion inverted to the Proportion given.

The second Effect is, by two Pieces, or Value of two several Pieces of Money and Merchandize known, to find how many Pieces of the one Price is to be given for so many of the other; and so to reduce and exchange one Sort of Money or Merchandize into another. Or else to find the Price unknown of any Price given to exchange in reciprocal Proportion.

The third Effect is, by two different Prices of a Measure of Wheat bought or sold, and the Weight of a Loaf of Bread, made answerable to one of the Prices of the Measure given, to find out the Weight of the same Loaf answerable to the other Price of the said Measure given.

Or else, by the two several Weights of the same priced Loaf, and the Price of the Measure of Wheat answerable to one of those Weights given, to find out the other Price of the Measure answerable to the other Weight of the same Loaf.

The fourth Effect is, by two Lengths and one Breadth of two rectangular Planes known, to find out another Breadth unknown. Or, by two Breadths and one Length given, to find out another Length unknown in an inverted Proportion.

The fifth Effect is, by a double Time and a capital Sum of Money borrowed or lent, to find out another capital Sum answerable to one of the given Times; or otherwise, by two capital Sums, and a Time answerable to one of them given, to find out a Time answerable to the other capital Sum in reciprocal Reason.

The sixth Effect is, by two different Weights of Carriage, and the Distance of the Place in Leagues or Miles given, to find another Distance in Miles answerable to the same Price of Payment. Or otherwise, by two Distances in Miles, and the Weight answerable to one of the Distances (being carried for a certain Price) to find out the Weight answerable to the other Distance for the same Price.

The seventh Effect is, by double Workmen, and the Time answerable to one of the Numbers of Workmen given, to find out the Time answerable to the other Number of Workmen, in the Performance of any Work or Service. Or contrarywise, by double Time, and the Workmen answerable to one of those Times given, to find out the Number of Workmen answerable to the other Time, in the Performance of any Work or Service.

Also by a double Price of Provision; and the Number of Men or other Creatures nourished for a certain Time, answerable to one of the prices of Provision given, to find out another Number of Men or other Creatures answerable to the other price of the Provision for the same Time. Or contrarywise, by two Numbers of Men or other Creatures nourished, and one price of Provision answerable to one of the Numbers of Creatures given, to find out the other price of the same Provision answerable to the other Number of Creatures both being supposed to be nourished for the same, &c.



To prove the Operation of the Rule of Three Inverse, multiply the 3d and 4th Terms together, and note their Product, and multiply the 1st and 2d together, and if their Product is equal to the Product of the 3d and 4th, then is the Work truly wrought, but if it falleth out otherwise, then it is erroneous.

As in the first Question of this Chapter, 16 (the third Number) being multiplied by 6 (the fourth Number) the Product is 96, and the Product of 8 (the first Number) multiplied by 12 (the second Number) is 96, equal to the first Product, which proves the Work to be right.

And note, 'That if in Division any Thing remain, such Remainder must be added to the Product of the third and fourth Terms, and if the Sum be equal to the Product of the first and second) the homogeneal Terms being of one Denomination, the Work is right.

## C H A P. XII.

### *The Double Rule of Three Direct.*

**W**E have already delivered the Rule of single Proportion, and we come now to lay down the Rules of plural Proportion.

1. Plural Proportion is, when more Operations in the Rule of Three than one are required before a Solution can be given to the Question propounded. Therefore in Questions that require Plurality in Proportion, there are always given more than three Numbers.

2. When there are given five Numbers, and a sixth is required in proportion thereunto, then the sixth Proportion is said to be found out by the Double Rule of Three, as in the Question following, viz.

If 100*l.* in 12 Months gain 6*l.* Interest, how much will 75*l.* gain in 9 Months?

3. Questions in the Double Rule of Three may be resolved either by two single Rules of Three, or by one single Rule of Three compounded of the five given Numbers.

4. The Double Rule of Three is either Direct or else Inverse

5. The Double Rule of Three Direct is, when unto 5 given Numbers, a 6th Proportional may be found out by two single Rules of Three Direct.

6. The 5 given Numbers in the Double Rule of Three Direct consist of two Parts, viz. 1. A Supposition, and 2dly, of a Demand: The Supposition is contained in the three first of the five given Numbers, and the Demand lies

in the two last, as in the Example of the second Rule of this Chapter, *viz.* If 100*l.* in 12 Months gain 6*l.* Interest, what will 75*l.* gain in 9 Months? Here the Supposition is expressed in 100, 12, and 6; for it is said, if 100*l.* in 12 Months gain 6*l.* Interest; And the Demand lieth in 75 and 9; for it is demanded, How much 75*l.* will gain in 9 Months.

7. When your Question is stated, the next Thing will be to dispose of the given Numbers in due Order and Place, as a preparative for Resolution; which that you may do, first, Observe which of the given Numbers in the Supposition is of the same Denomination with the Number required, for that must be the 2<sup>d</sup> Number (in the first Operation) of the Single Rule of Three, and one of the other Numbers in the Supposition (it matters not which) must be the first Number, and that Number in the Demand, which is of the same Denomination with the first, must be the third Number; which three Numbers being thus placed, will make one perfect Question in the Single Rule of Three, as in the forementioned Example; first, I consider, that the Number required in the Question, is in the Interest or Gain of 75*l.* therefore that Number in the Supposition which hath the same Name, *viz.* 6*l.* which is the Interest or Gain of 100*l.* must be the second Number in the first Operation, and either 100 or 12 (it matters not which) must be the first Number, but I will take 100; and then for the third Number, I put that Number in the Demand which hath the same Denomination with 100, which is 75, for they both signify Pounds principal, and then the Numbers will stand as you see in the Margent.

But if I had for the first Number put the other Number in the Supposition, *viz.* 12, which signifies 12 Months, then the third Number must have been 9, which is the Number in the Demand which hath the same Denomination with the first, *viz.* 9 Months, and they will stand as in the Margent,

There yet remains two Numbers to be disposed of, and those are one in the Supposition, and another in the Demand; that which is of the Supposition, I place under the first of the three Numbers; and the other, which is the Demand, I place under the third Number; and then two of the Terms in the Supposition will stand (one over the other) in the first place, and the two Terms in the Demand will stand (one over the other) in the third place, as in the Margent.

r. Having disposed or ordered the given Numbers according to the last Rule, we may proceed to a Resolution: And first I work with the 3 uppermost Numbers, which, according to the first Disposition are 100 : 6 :: and 75; which is as much as to say, if 100*l.* requires 6*l.* Interest, how much will 75*l.* require? Which, by the third Rule of the 11th Chapter, I find to be Direct, and by the 7th and 8th Rules of the 10th Chapter, I find the 4th proportional Number to be 4*l.* 10*s.* so that by the foregoing single Question I have discovered how much Interest 75*l.* will gain in 12 Months; the Operation whereof followeth on the left Hand under the Letter A: And having discovered how much it will gain in 12 Months, we may by another Question easily discover how much it will gain in 9 Months; for this 4th Number (thus found) I put in the Middle between the two lowest Numbers of the 5, after they are placed according to the 7th Rule of this Chapter, and then it will be a second Number, in another Question of the Rule

M.      l.      s.      M.

of Three. The Numbers being 12 : 4 10 :: 9 the first and third Numbers being of one Denomination, viz. both Months, and may be thus expressed; if 12 Months require 4*l.* 10*s.* Interest, what will 9 Months require? And by the third Rule of the 11th Chapter, I find it to be the Direct Rule, and by working according to the Directions laid down in the 7th, 8th and 9th Rules of the 10th Chapter, I find the fourth proportional Number to the last single Question to be 3*l.* 7*s.* 6*d.* which is the sixth proportional Number to the five given Numbers, and is the Answer to the general Question. The Work of the last single Question is expressed on the right Side of the Page, under the Letter B, as followeth.

A

100  
12  
1. 1. 1.  
If 100 6 75  
75  
30  
42  
100) 4 50 (4 10  
4  
Rem. (50)  
Mult. 20  
100) 10 00 (10s.  
1. s.  
Facit 4 10

—6—75

9

Then say,

M. l. s. M.

If 12 4 10 9

20

90

12

180

90

1080 Pence

9

-----12 20 l. s. d.

12) 9720 (810 67 (3 7 6

96

72

-----

7 s.

12

90

12

84

(0)

(6) d.

Facit 3l. 7s. 6d.

B

So that by the foregoing Operation I conclude, that if, 100l. in 12 Months, gain 6l. Interest, 75l. will gain 3l. 7s. 6d. in 9 Months, after the same Rate. The Answer would have been the same if the given Numbers had been ordered according to the second Method, viz. as you see in the Margin.

For first, I say, if 12 Months gain 6l. what will 9 Months gain? This Question I find to be direct, by the 3d Rule of the 11th Chapter, and by the 7th and 8th Rules of the 10th Chapter, I find the fourth proportional Number to these three to be 4l. 10s.

Thus have I found out what is the Interest of 100l. for 9 Months, and am now to find the Interest of 75l. for 9 Months; to effect which, I make this fourth Number found as before) to be my second Number in the next Question, I say, if 100l. require 4l. 10s. what will 75l. require? This Question I find (by the said 3d Rule of the 11th Chapter) to be direct, and by the said 7th, 8th, and 9th Rules of the 10th Chapter, I find the Answer to be as before, viz. 3l. 7s. 6d.

The Operation of this Rule in the following Questions, is purposely omitted, to try the Learner's Capacity.

Quest.

*Quest. 2.* A second Example in this Rule may be as followeth, viz. A Carrier receiving 42s. for the Carriage of 3 C. Weight 150 Miles, I demand how much he ought to receive for the Carriage of 7 C. 3 qrs. 14 lb 50 Miles at that Rate? *Answer*, 36s. 9d.

*Quest. 3.* A Regiment of 936 Soldiers eat up 351 Quarters of Wheat in 168 Days, I demand how many Quarters of Wheat 11232 Soldiers will eat in 56 Days at that Rate? *Answer*, 1404 Quarters.

*Quest. 4.* If 40 Acres of Grass be mowed by 8 Men in 7 Days, how many Acres shall be mowed by 24 Men in 28 Day? *Answer*, 480 Acres.

*Quest. 5.* If 48 Bushels of Corn (or other Seed) yield 576 Bushels in a Year, how much will 240 Bushels yield in 6 Years at that Rate? That is to say, if there were sowed 240 Bushels every one of the 6 Years? *Answer*, 17280 Bushels.

*Quest. 6.* If 40 Shillings be the Wages of 8 Men for 5 Days, what will be the Wages of 32 Men for 24 Days? *Answer*, 768 Shillings, or 38l. 8s.

*Quest. 7.* If 14 Horses eat 56 Bushels of Provender in 16 Days, how many Bushels will 20 Horses eat in 24 Days? *Answer*, 120 Bushels.

*Quest. 8.* If 8 Cannons in one Day spend 48 Barrels of Powder, I demand how many Barrels 24 Cannons will spend in 12 Days at that Rate? *Answer*, 1728 Barrels.

*Quest. 9.* If in a Family consisting of 7 Persons, there are drank out 2 Kilderkins of Beer in 12 Days, how many Kilderkins will there be drank out in 8 Days, by another Family consisting of 14 Persons? *Answer*, 48 Gallons, or 2 Kilderkins and 12 Gallons.

*Quest. 10.* An Usurer put 75l. out, to receive Interest for the same, and when it had continued 9 Months, he received for Principal and Interest 78l. 7s. 6d. I demand at what Rate per Cent. per Annum he received Interest? *Answer*, 6l. per Cent. per Annum.

## C H - A P. XIII.

### The Double Rule of Three Inverse.

**T**HE Double Rule of Three Inverse is, when a Question in the Double Rule of Three is resolved by two single Rules of Three, and one of those single Rules falls out to be Inverse, or requires a fourth Number in Proportion reciprocal (for both Questions are never Inverse.)



2. In all Questions of the Double Rule of Three (as well Inverse as Direct) you are, in the disposing of the 5 given Numbers, to observe the 7th Rule of the 12th Chap. and in resolving of it by two single Rules, observe to make choice of your Numbers for the first and second single Questions according to the Directions given in the 8th Rule of the same Chapter, and in the Example following, viz.

*Quest.* 1. If 100*l.* Principal in 12 Month gain 6*l.* Interest, what Principal will gain 3*l.* 7*s.* 6*d.* in 9 Months?

This Question is an Inversion of the first Question of the 12th Chapter, and may serve for a Proof thereof.

In order to a Resolution, I dispose of the 5 given Numbers according to the 7th Rule of the last Chapter; and being so disposed, they will stand as follow.

12	100	9
6		3 7 6

Or thus,

<i>l.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>
6	100	3	7	6
12		9		

Here observe, That according to the 8th Rule of the 12th Chapter, the first Question (if you take it from the 5 Numbers, as they are ordered or placed first) will be, if 12 Months require 100*l.* Principal, what will 9 Months require to make the same Interest? This (according to the 3d Rule of the 11th Chapter) is Inverse, and the Answer will be found (by the 2d Rule of the 11th Chapter) to be 133*l.* 6*s.* 8*d.* The second Question then will be, if 6*l.* Interest require 133*l.* 6*s.* 8*d.* Principal; how much Principal will 3*l.* 7*s.* 6*d.* require? This is a direct Rule, and the Answer in a direct Proportion is 75*l.* See the Work.

First I say,

<i>M.</i>	<i>l.</i>	<i>M.</i>
12	100	9

	12	
9)	1200	(133 6 8
	..	

9	<i>l.</i>	<i>s.</i>	<i>d.</i>
30	Facit	133	6 8
27			

30	
27	
(3)	
20	
9)	60 (6 <i>s.</i>
54	
(6)	
12	
9)	72 (8 <i>d.</i>
72	
(0)	

Then

Then I say,

<i>l.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
If 6 :	133	6	8	::	3	7 6
240	20				20	
1440 <i>d.</i>	2666				67	
	12				12	
	5340				140	
	2666				67	
	32000				810 <i>d.</i>	
	810					
	320000					
	256					

-----2 10  
 144|0 2492000,0 (1800|0 (75*l.*  
 ....

144	168
1152	120
1152	120
(0)	(0)

So that by the foregoing Work I find that if 6*l.* Interest be gained by 100*l.* in 12 Months, 3*l.* 7*s.* 6*d.* will be gained by 75*l.* in 9 Months.

But if the Resolution had been found out by the Numbers as they are ranked in the second Place, then the second Question in the second Rule would have been Inverse, and the first Question Direct, and the Conclusion the same with the first Method, *viz.* 75*l.*

*Quest. 2.* If a Regiment consisting of 936 Soldiers can eat up 351 Quarters of Wheat in 168 Days, how many Soldiers will eat up 1400 Quarters in 56 Days, at that Rate? *Answer* 11200 Soldiers.

*Quest. 3.* If 12 Students in 8 Weeks spend 48*l.* I demand how many Students will spend 288*l.* in 18 Weeks? *Answer* 32 Students.

*Quest. 4.* If 48*l.* serve 12 Students 8 Weeks, how many Weeks will 288*l.* serve 4 Students? *Answer* 144 Weeks.

*Quest. 5.* If when a Bushel of Wheat cost 3*s.* 4*d.* the Penny Loaf weighing 12 Ounces, I demand the Weight of the Loaf worth 9*d.* when the Bushel cost 10*s.* *Answer* 35-oz.

*Quest. 6.* If 48 Pioneers in 12 Days cast a Trench 24 Yards long, how many Pioneers will cast a Trench 168 Yards long in 16 Days? *Answer* 252 Pioneers.

*Quest. 7.* If 12 C.wt being carried 100 Miles, cost 5*l.* 12*s.* I desire to know how many C.wt may be carried 150 Miles for 12*l.* 12*s.* at that Rate? *Ans.* 18 C.

*Quest.*

*Quest. 8.* If when Wine is worth 30*l.* per Ton, 20*l.* worth is sufficient for the Ordinary of 100 Men, how many Men will 4*l.* worth suffice when it is worth 24*l.* per Ton? *Answer,* 25 Men.

*Quest. 9.* If 6 Men in 24 Days mow 72 Acres, in how many Days will 8 Men mow 24 Acres? *Answer,* in 6 Days.

*Quest. 10.* If when the Ton of Wine is worth 30*l.* 100 Men will be satisfied with 20*l.* worth, I desire to know what the Ton is worth when 4*l.* worth will satisfy 25 Men at the same Rate? *Answer,* 24*l.* per Ton.

## CHAP. XIV.

### *The Rule of Three composed of five Numbers.*

**T**HE Rule of Three composed is, when Questions (wherein there are five Numbers given, to find a sixth in proportion thereunto) are resolved by one single Rule of Three composed of five given Numbers.

2. When Questions may be performed by the Double Rule of Three Direct, and it is required to resolve them by the Rule of Three composed; first order or rank your Numbers according to the 7th Rule of the 12th Chap. then  
*The Rule is,*

Multiply the Terms or Numbers (that stand one over the other in the first Place) the one by the other, and make their Product the first Term in the Rule of Three Direct; then multiply the Terms that stand one over the other in the third Place, and Place their Product for the third Term in the Rule of Three Direct, and put the middle Term of the 3 uppermost for a second Term; then having found a fourth Proportional direct to these three, this fourth Proportional so found shall be the Answer required.

So the first Question of the 13th Chapter being proposed, viz. if 100*l.* in 12 Months gain 6*l.* Interest, what will 75*l.* gain in 9 Months?

The Numbers being ranged or placed as is there directed and done, then I multiply the two first Terms 100 and 12 the one by the other, and their Product is 1200 for the first Term; then I multiply the last two Terms 75 and 9 together, and their Product is 675 for the third Term; Then I say, as 1200 is to 6, so is 675 to the Answer, which by the Rule of Three Direct will be found to be 3*l.* 7*s.* 6*d.* as was before found.

3. But if the Question is to be answer'd by the Double Rule of Three Inverse, then (having placed the 5 given Terms as before) multiply the lowermost Term of the first Place

Place by the uppermost Term of the third Place, and put the Product for the first Term; then multiply the uppermost Term of the first Place by the lowermost Term of the third Place, and put the Product for the third Term, and the second Term of the three highest Numbers for the middle Term of those two; then if the Inverse Proportion is found in the uppermost three Numbers, the fourth Proportional direct to these three will be the Answer. So the first Question in the 13th Chapter being stated, *viz.* if 100 Principal in 12 Months gain 6*l.* Interest, what Principal will gain 3*l.* 7*s.* 6*d.* in 9 Months? State the Numbers as there directed in the first Order, *viz.*

M.	l.	M.
12	100	9
l.		l. s. d.
6		3 7 9

Then reduce the 6*l.* and 3*l.* 7*s.* 6*d.* into Pence, the 6*l.* is 144*od.* and 3*l.* 7*s.* 6*d.* is 810*d.* then multiply 1440 by 9, the Product is 12960 for the first Term in the Rule of Three Direct, and multiply 810 by 12, the Product is 9720 for the third Term; then I say, as 12960 is to 100*l.* so is 9720 to the Answer, *viz.* 75*l.* as before. But if the Terms had been placed after the second Order, *viz.*

l.	l.	l.	s.	d.
6	:	100	:	3 7 6
M.		M.		
12		9		

Then the Inverse Proportion is found in the lowest Numbers, and having composed the Numbers for a single Rule of Three, as in the second Rule foregoing; then the Answer must be found by a single Rule of Three Inverse; for here it falls out to multiply 810 by 12 for the first Number, 1440 by 9 for the third Number; and then you must say, as 9720 is to 100*l.* so is 12960 to the Answer, which by Inverse Proportion will be found to be 75*l.* as before.

The Question in the 12th and 13th Chapters may serve for thy farther Experience.

## C H A P. XV.

### Single Fellowship.

**F**ellowship is that Rule of Plural Proportion whereby we ballance Accompts depending between diverse Persons, having put together a general Stock, so that they may every Man have his proportional Part of Gain, or sustain his proportional Part of Loss.

2. The Rule of Fellowship is either single, or it is double.

3. The single Rule is, when the Stocks propounded are single Numbers, without any respect or relation to Time, each Partner continuing his Money in Stock for the same time.

4. In the single Rule of Fellowship the Proportion is, as the whole Stock of all the Partners is in proportion to the total Gain or Loss, so is each Man's particular Share in the Stock, to his particular Share in the Gain or Loss. Therefore take the Total of all the Stocks for the first Term in the Rule of Three, and the whole Gain or Loss for the second Term, and the particular Stock of any one of the Partners for the third Term, then multiply and divide according to the seventh Rule of the 9th Chapter, and the fourth proportional Number is the particular Loss or Gain of him whose Stock you made your second Number, wherefore repeat the Rule of Three as often as there are particular Stocks or Partners in the Question, and the fourth Terms produced upon the several Operations are the respective Gain or Loss of those particular Stocks given, as in the example following.

*Quest. 1.* Two Persons, viz. *A* and *B*, bought a Tun of Wine, for 20*l.* of which *A* paid 12*l.* and *B* paid 8*l.* and they gained in the Sale thereof 5*l.* now I demand each Man's Share in the Gain, according to his Stock?

First, I find the Sum of all their Stocks, by adding them together, viz. 12*l.* and 8*l.* which are 20*l.* then according to this Rule, I say first, if 20*l.* (the Sum of their Stocks) require 5*l.* the total Gain, how much will 12*l.* (the Stock of *A*) require? Multiply and divide by the 7th Rule of the 9th Chapter, and the Answer is 3*l.* for the Share of *A* in the Gain; then again say, if 20*l.* require 5*l.* what will 8*l.* require? The Answer is 2*l.* which is the Gain of *B*; so I conclude the Share of *A* in the Gain is 3*l.* and the Share of *B* in the Gain is 2*l.* which in all is 5*l.*

$$\begin{array}{r} \text{If } 20 : 5 :: 12 \end{array}$$

$$\begin{array}{r} 12 \\ 20 \overline{) 60} \quad (3 \text{ l.} \\ \underline{60} \\ (0) \end{array}$$

$$\begin{array}{r} \text{If } 20 : 5 :: 8 \end{array}$$

$$\begin{array}{r} 8 \\ 20 \overline{) 40} \quad (2 \text{ l.} \end{array}$$

*Quest.*



*Quest. 2.* Three Merchants, viz. *A, B* and *C*, enter upon a joint Adventure, *A* put into the common Stock 78*l.* put in 117*l.* and *C* put in 234*l.* and they find (when they make up their Accounts) that they have gained in 264*l.* now I desire to know each Man's particular Share of the Gain?

First, I add their particular Stocks together, and the Sum is 429*l.* then say, if 429*l.* gain 264*l.* what will 78*l.* gain? And what will 117*l.* and what will 234*l.* (the Stock, of *A, B* and *C*) gain? Work by three several Rules of Three, and you will find that

$$\begin{array}{rcl} \text{The Gain of } \left\{ \begin{array}{l} A \\ B \\ C \end{array} \right\} & \text{is } \left\{ \begin{array}{l} 48 \\ 72 \\ 114 \end{array} \right\} \\ & \text{Sum } 264 \end{array}$$

*Quest. 3.* Four Partners, viz. *A, B, C* and *D*, among them built a Ship, which cost 1730*l.* of which *A* paid 340*l.* *B* 519*l.* *C* 692*l.* and *D* 173*l.* and her Freight for a certain Voyage is 370*l.* which is due to the Owners or Builders; demand each Man's Share therein, according to his Character in building her?

$$\begin{array}{rcl} \text{Answer, } A & \left\{ \begin{array}{l} 74 \\ 111 \\ 148 \\ 37 \end{array} \right\} \\ B & \\ C & \\ D & \\ \hline & 370 \end{array}$$

*Quest. 4.* *A, B* and *C* inter into Partnership for a certain Time, *A* put into a common Stock 364*l.* *B* put in 480*l.* *C* put in 500*l.* and they gained 867*l.* now I demand each Man's Share in the Gain, proportionable to his Stock?

$$\begin{array}{rcl} A & \left\{ \begin{array}{l} 234 \\ 310 \\ 222 \end{array} \right\} & \begin{array}{l} 9 \\ 9 \\ 1 \end{array} \\ B & \\ C & \\ \hline \text{Sum } 867 & 0 & 0 \end{array}$$

5. To prove the single Rule of Fellowship add each Man's particular Gain or Loss together, and if the total Sum is equal to the general Gain or Loss, then is the Work rightly performed, but otherwise it is erroneous. *Example*; In the first Question of this Chapter, the Answer was, That the Gain of *A* was 3*l.* and the Gain of *B* 2*l.* which added together makes 5*l.* equal to the total Gain given.

If in finding out the particular Shares of the several Partners

*The Proof of the Rule of Single Fellowship*

Partners, any Thing remaining after Division is ended, such Remainders must be added together, (they being all Fractions of the same Denomination) and their Sum divided by the common Divisor in each Question, viz. the total Stock, and the Quotient added to the particular Gains; and then if the total Sum is equal to the total Gain, the Work is right, otherwise not.

As in the 4th Question, the Remainders were 354, 62 and 930, which added together make 1346, which divided by 1346 (the Sum of their Stocks) the Quotient is 1*d*. which I add to the Pence, &c. and the Sum of their Shares is 897*l*. equal to the total Gain, wherefore I conclude the Work is right.

## C H A P. XVI.

## Double Fellowship.

**D**ouble Fellowship is, when several Persons enter into Partnership for unequal Time; that is, when every Man's particular Stock hath relation to a particular Time.

2. In the Double Rule of Fellowship, multiply each particular Stock by its respective Time, and having added the several Products together, make their Sum the first Number (or Term) in the *Rule of Three*, and the total Gain or Loss the second Number, and the Product of any one's particular Stock by his Time the third Term, and the fourth Number in proportion thereunto is his particular Gain or Loss, whose Product of Stock and Time is your third Number.

Then repeat (as in *Single Fellowship*) the *Rule of Three*, as often as there are Products (or Partners) and the four Terms thereby invented, are the Numbers required.

## Example.

*Quest. 1.* *A* and *B* enter Partnership; *A* put in 40*l*. for 3 Months, *B* put in 75*l*. for 4 Months, and they gained 70*l*. now I demand each Man's Share in the Gain, proportional to his Stock and Time? *Answer, A* 20*l.* *B* 50*l.*

To resolve this Question, I first multiply the Stock of *A*, (viz. 40*l.*) by its Time (3 Months) and the Product is 120; then I multiply the Stock of *B* by its Time, viz. 75*l.* by 4, and it produceth 300, which I add to the Product of *A*, his Stock and Time, and the Sum is 420. Then by the Rule of Three Direct I say, as 420 (the Sum of the Products) is to 70, (the total Gain) so is 120 the Product of *A*, his Stock and Time) to 20*l.* (the Share of

	<i>l.</i>	<i>l.</i>
	40	75
	3	4
<i>A</i>	120	<i>B</i> 300
		120
		Sum 420
		of

of *A* in the Gains) Then I say again, as 420 is to 70, so is 300 (the Product of *B* his Stock and Time) to 50*l.* (the Share of *B* in the Gains: And that each is to have for his Share.

*Quest. 2.* *A*, *B* and *C* make a Stock for 12 Months, *A* put in at first 364*l.* and 4 Months after that he put in 40*l.* *B* put in at first 408*l.* and at the End of the 7 Months he took out 86*l.* *C* put in at first 148*l.* and 3 Months after he put in 86*l.* more, and 5 Months after that he put in 100*l.* more, and at the End of 12 Months their Gain is found to be 1436*l.* I desire to know each Man's Share in the Gains, according to his Stock and Time?

First, I consider that the whole Time of their Partnership is 12 Months: Then I proceed to find out the several Products, of Stock and Time, as followeth:

*A* had at first 364*l.* for 4 Months, wherefore that Product is

Then he put in 40*l.* which with the first Sum makes 404*l.* which continued the Remainder of the Time, viz. 8 Months and that Product is

The Sum of the Products of the Stock and Time of *A* is

*B* had 408*l.* in 7 Months, whose Product is

And then took out 86*l.* therefore he left in Stock 322*l.* which continued the rest of the Time, viz. 5 Months, whose Product is

The Sum of the Products of the Stock and Time of *B* is

*C* put in 148*l.* for 3 Months, whose product being multiplied by 3, is

Then he put in 86*l.* which added to the first, (viz. 148*l.*) makes 234*l.* which lay in Stock 5 Months, and their Product is

Then he put in 100*l.* more, so then he had in Stock 334*l.* which continued the Remainder of the Time, 4 Months, which multiplied together, produces

The Sum of the Product of the Money and Time of *C* is

*B* --- --- --- ---

*A* --- --- --- ---

The total Sum of all the Products is

Then I say, as 12104 is to 1436 (the total Gain) so is 4688 to the Share of *A* in the total Gain, &c. go on as in the foregoing Examples, and you will find their Shares in the Gain to be as followeth, viz.

Answer

*Answer,*

$$\text{The Share of } \left\{ \begin{array}{l} A \\ B \\ C \end{array} \right\} \text{ is } \left\{ \begin{array}{l} 556 \quad 03 \quad 6 \quad \overset{6192}{12104} \\ 529 \quad 16 \quad 9 \quad \overset{5496}{12104} \\ 349 \quad 19 \quad 8 \quad \overset{416}{12104} \end{array} \right.$$


---


$$1436 \quad 00 \quad 0$$

*Quest. 3.* Three Grasers, *A*, *B* and *C*, take a Piece of Ground for 46*l.* 10*s.* in which *A* put 12 Oxen for 8 Months, *B* put in 16 Oxen for 5 Months, and *C* put 18 Oxen for 4 Months; now the Question is, what each Man shall pay of the 46*l.* 10*s.* for his Share in that Charge?

$$\left. \begin{array}{l} A \\ B \\ C \end{array} \right\} \text{ shall pay } \left\{ \begin{array}{l} 18 \quad 00 \\ 15 \quad 00 \\ 13 \quad 10 \end{array} \right.$$


---


$$46 \quad 10$$

3. The Proof of this Rule is the same with that of Single Fellowship, laid down in the 5th Rule of the 15th Chapter; and note, that

If a Loss be sustained instead of a Gain among Partners, every Man's Share to be born in the Loss, is to be found after the same Method as their Gain, whether their Stocks be for equal or unequal Time.

## CHAP. XVII.

### *Alligation Medial.*

1. **T**HE Rule of Alligation is that Rule in plural Proportion, by which we resolve Questions wherein is a Composition or Mixture of diverse Simples, as also it is useful in Composition of Medicines, both for Quantity, Quality or Price: And its Species are two, *viz.* Medial and Alternate.

2. Alligation Medial is, when having the several Quantities and Prices of several Simples propounded, we discover the mean Price or Rate of any Quantity of the Mixture compounded of those Simples, and the Proportion is,

As the Sum of the Simples to be mingled is to the total Value of all the Simples, so is any Part or Quantity of the Composition or Mixture to its mean Rate or Price.

*Quest. 1.* A Farmer mingled 20 Bushels of Wheat, at 5*s.* per Bushel, and 36 Bushels of Rye at 3*s.* per Bushel, with 40 Bushels of Barley at 2*s.* per Bushel; now I desire to know what one Bushel of that Mixture is worth?

To resolve this Question, add together the given Quantities and their Value, which is 96 Bushels, whose total Value is 14*l.* 8*s.* as appeareth by the Work following; for

Bushels

Bushels

20 of Wheat, at 5s. per Bushel,	is	5 0
36 of Rye, at 3s. per Bushel,	is	5 8
40 of Barley, at 2s. per Bushel,	is	4 0

The Sum of their given Quantities is 96, and }  
 their Value is } 14 8

Then say, by the Rule of Three Direct, if 96 Bushels  
 cost or is worth 14l. 8s. what is one Bushel worth?

Bush.	1.	s.	Bush.
96	14	8	1 Bushel
	20		
96)	288	(3 s.	

288 Facit 3s. per Bushel.  
 (0)

Quest. 2. A Vintner mingled 15 Gallons of Canary at 8s. per Gallon, with 20 Gallons of Malaga at 7s. 6d. per Gallon, with 10 Gallons of Malaga at 6s. 4d. per Gallon, and 24 Gallons of White-wine at 4s. per Gallon, now I demand what a Gallon of this Mixture is worth? Work as in the last Question, and you will find the Answer to be 6s. 2d. 2qrs.  $\frac{4}{9}$ .

Quest. 3. A Grocer hath mingled 3 C. of Sugar at 56s. per C. with 4 C. of Sugar at 3l. 14s. 8d. per C. and with 6 C. at 1l. 17s. 4d. per C. I desire to know the Price of a C. wt. of that Mixture? Answer, 2l. 13s. 1d.  $\frac{7}{12}$ .

3. The Proof of this Operation is by the Price of any Quantity of the Mixture, to find out the total Value of the whole Composition, and if it is equal to the total Value of the several Simples, the Work is right, otherwise not. (*The Proof of Alligation Medial.*) As in the first Example, the Answer to the Question was that 3s. is the Price of 1 Bushel; wherefore I say, by the Rule of Proportion, if 1 Bushel be 3s. what is 96 Bushels? Answer, 14l. 8s. which is the total Value of the several Simples; wherefore the Work is right.

## C H A P. XVIII.

## Alligation A'ternate.

1. **A**lligation Alternate is, when there are given the particular Prices of several Simples, and thereby we discover such Quantities of those Simples, as being mingled together, shall bear a certain Rate propounded.

2. When



8.  
1.  
0  
8  
0  
1  
hels  
y at  
per  
lon,  
w l  
ork  
to be  
56.  
with  
of a  
any  
e of  
alue  
not.  
ple,  
of 1  
if 1  
h is  
the

2. When such a Question is stated, place the given Prices of the Simples one over the other, and the propounded Price of the Composition against them in such Sort that it may represent a Root, and they as so many Branches springing from it, as in the following Example.

*Quest 1.* A certain Farmer is desirous to mix 20 Bushels of Wheat at 5s. or 60d. per Bushel, with Rye at 3s. or 36d. per Bushel, and with Barley at 2s. or 24d. per Bushel, and Oats at 1s. 6d. per Bushel, and desireth to mix such a Quantity of Rye, Barley and Oats, with the 20 Bushels of Wheat, as that the whole Composition may be worth 2s. or 32d. per Bushel.

The Prices of the Simples being placed according to the last Rule (with the Price of the Composition propounded as a Root to them) will stand as followeth.

$$\begin{array}{r} 60 \text{ Pence} \\ 32 \left\{ \begin{array}{l} 36 \\ 24 \\ 18 \end{array} \right. \end{array}$$

3. Having thus placed the given Numbers, you are to link the several Rates of the Simples one to the other, by certain Arches, in such sort that one that is lesser than the mean Rate, may be coupled to another that is greater than the mean Rate; so the Question last propounded will stand,

1. Thus,

2. Or thus,

3. Or thus,

$$32 \left\{ \begin{array}{l} 60 \\ 36 \\ 24 \\ 18 \end{array} \right. \curvearrowright$$

$$32 \left\{ \begin{array}{l} 60 \\ 36 \\ 24 \\ 18 \end{array} \right. \curvearrowright$$

$$32 \left\{ \begin{array}{l} 60 \\ 36 \\ 24 \\ 18 \end{array} \right. \curvearrowright$$

4. Then take the Difference between the Root and the several Branches, and place the Difference of each against the Number or Branch with which it is coupled or linked, and having taken all the Differences and placed them as aforesaid, then those Differences so placed will shew you the Number of each Simple to be taken to make a Composition to bear the mean Rate propounded.

So the Branches of the last Question being linked together, as in the first manner, I say, the Difference between 2 and 60 is 28, which I put against 18, because 60 is linked with 18; then the Difference between 32 and 36 is 4, which I put against 24, because 36 is linked or coupled with 24; then I say the Difference between 32 and 24 is 8, which I placed against 36 (for the Reason aforesaid) then I say, the Difference between 32 and 18 is 14, which I place against 60, and then the Work will stand as you see in the Margent.

$$\begin{array}{r} 60 \\ 32 \left\{ \begin{array}{l} 36 \\ 24 \\ 18 \end{array} \right. \curvearrowright \begin{array}{l} 14 \\ 8 \\ 4 \\ 28 \end{array} \end{array}$$

So

So I conclude that a Composition made of 14 Bushels of Wheat at 60*s.* *per* Bushel, and 8 Bushels of Rye at 36 *per* Bushel, and 4 Bushels of Barley at 24*d.* *per* Bushel, and 28 Bushels of Oats at 18*d.* *per* Bushel, will bear the mean Price of 32*d.* or 2*s.* 8*d.* *per* Bushel. And here observe, that in the Composition there is but 14 Bushels of Wheat, but I would mingle 20 Bushels; and this Kind (or rather Case) of Alligation Alternate, *viz.* when there is given a certain Quantity of one of the Simples, and the Quantities of the rest sought to mingle with the given Quantity, that the Whole may bear a Price propounded) is called *Alternatio partial.*

And the Proportion to find out the several Quantities to be mingled with the given Quantity, is thus,

As the Difference annexed to the Branch, that is, the Value of an Integer of the given Quantity, is to the other particular Differences, so is the Quantity given to the several Quantities required.

So here, to find how much Rye, Barley and Oats may be mingled with the 20 Bushels of Wheat, I say, by the Rule of Three Direct, if 14 Bushels of Wheat require 8 Bushels of Rye, what will 20 Bushels of Wheat require? *Answer*, 11  $\frac{6}{7}$  Bushels of Rye.

Again, if 14 Bushels of Wheat require 4 Bushels of Barley, what will 20 Bushels of Wheat require? *Ans.* 5  $\frac{1}{2}$  Bushels of Barley. Again, I say, if 14 Bushels of Wheat require 28 Bushels of Oats what will 20 Bushels of Wheat require? *Ans.* 40 Bushels of Oats.

And now I say, that 20 Bushels of Wheat mingled with 11  $\frac{6}{7}$  Bushels of Rye, and 5  $\frac{1}{2}$  Bushels of Barley, and 40 Bushels of Oats, each bearing the Rate as aforesaid, will make a Composition, or Heap of Corn, that may yield 32*d.* *per* Bushel.

But if the Branches had been coupled according to the second Order or Manner, the Differences would have been

thus placed, *viz.* the Difference between 32 and 60 is 28, which I set against 8; the Difference between 32 and 36 is 4, which I set against 14; the Difference between 32 and 24 is 8, which I set against 4; the Difference between 32 and 18 is 14, which I set against 28.

18, which I set against 60; then the Difference between 60 and 18 is 42, which I set against his Yoke-fellow 36; then I conclude, that if you mix'd 8 Bushels of Wheat with 14 Bushels of Rye, 28 Bushels of Barley, and 40 Bushels of Oats, each bearing the aforesaid Prices,

whole Mixture may be sold for 32*d.* per Bushel, as by the Work in the Margent.

You see by this Work we have found how many Bushels of Rye, Barley and Oats ought to be mixed with 8 Bushels of Wheat, and to find out how many of each ought to be mixed with 20 Bushels of Wheat, I say, as 8 is to 14, so is 20 to 35 Bushels of Rye. As 8 is to 28, so is 20 to 70 Bushels of Barley, as 8 is to 4, so is 20 to 10 Bushels of Oats; whereby I conclude, that if 20 Bushels of Wheat I put 35 Bushels of Rye, 70 Bushels of Barley, and 10 Bushels of Oats, each bearing the aforesaid Price per Bushel, that then a Bushel of this Mixture will be worth 32*d.* or *s.* 8*d.*

And if the Branches had been linked as you see in the 3*d* Place, where each Branch bigger than the Root is link'd to two that are lesser than the Root, then in this Case you must have placed the several Differences between the Root and Branches against those two with which each is coupled; as first, the Difference between 32 and 60 is 28, which I set against 24 and 18, because it is coupled with them both;

32	{	60	D)	8	14	22
		36		8	14	22
		24		28	4	32
		18		28	4	32

then the Difference between 32 and 36 is 4, which I set likewise against 24 and 18, because 36 is linked to them both; then the Difference between 32 and 24 is 8, which I put against 60 and 36, because 24 is linked to them both, then the Difference between 32 and 18 is 14, which I put against 60 and 36, the Yoke-follows of 18.

Lastly, I draw a Line behind the Differences, and add the Differences which stand against each Branch, and put the Sum behind the said Line against its proper Branch, as you see in the Margent.

And now by this Work I find that 22 Bushels of Wheat mingled with 22 Bushels of Rye, and 32 Bushels of Barley, and 32 Bushels of Oats, each bearing the said Price, will make a Mixture bearing the mean Rate of 32*d.* per Bushel.

And now to find how much of each of the rest must be mingled with 20 Bushels of Wheat, I say,

As 22 is to 22, so is 20 to 20 Bushels of Rye. As 22 is to 32, so is 20 to 29  $\frac{2}{3}$  Bushels of Barley. As 22 is to 32, so is 20 to 29  $\frac{2}{3}$  Bushels of Oats.

Wherefore you see the Questions of Alligation Alternate will admit of more true Answers than one; for we have found three several Answers to this first Question.

## The Proof of Alligation partial.

Questions of Alligation partial are proved the same way with Questions in Alligation medial, which you may see in the 3d Rule of the 17th Chapter.

*Quest. 3.* A Grocer hath 4 Sorts of Sugar, viz. of 12d. per lb., of 10d. per lb., of 6d. per lb., and of 4d. per lb., and would have a Composition worth 8d. per lb., the whole Quantity whereof should contain 144 lb. made of these four Sorts? I demand how much of each he must take?

Questions of this Nature are resolved by that Part of Alligation Alternate, called by Arithmeticians *Alligation Total*, viz. where there is given the Sum and Prices of several Simples, to find out how much of each Simple ought to be taken to make the said Sum or Quantity, so that it may bear a certain Rate propounded.

To resolve this Question, I place the several Prices of the Simples and mean Rate propounded, and link them together, as is directed in the 2d and 3d Rules of this Chapter, and place the Differences between the Root and Branches, according to the 4th Rule of this Chapter, which will then stand one of these three Ways, viz.

First		Second	
12	4	12	2
10	2	10	4
6	2	6	4
4	4	4	2
<hr/>		<hr/>	
12		12	
Third			
12	2, 4	6	
10	2, 4	6	
6	4, 2	6	
4	4, 2	6	
<hr/>		<hr/>	
		24	

5. Then add the several Differences together, which have done, and the Sums of the first and second Order are 12 lb. and of the third 24 lb. as you see above. But it is required that there should be 144 lb. of the Composition, therefore to find the Quantity of each Simple to make the whole Composition 144 lb., observe this general Rule viz.

As the Sum of the Differences is to the several Differences, so is the total Quantity of the Composition to the Quantity of each Simple.

So to find how much of each Sort of Sugar I ought to take to make 144lb at 8d. per lb.

As 12 is to 4, so is 144 to 48lb at 12d. per lb.

As 12 is to 2, so is 144 to 24lb at 10d. per lb.

As 12 is to 2, so is 144 to 24lb at 6d. per lb.

As 12 is to 4, so is 144 to 48lb at 4d. per lb.

Whereby I find that 48lb. at 12d. per lb, and 24lb at 10d. per lb, and 24lb at 6d. per lb, and 48lb at 4d. per lb, will make a Composition of Sugar containing 144 lb, worth 8d. per lb.

But as the Branches are linked in the second Order, the Answer will be 24lb at 12d. per lb, and 48lb at 10d. per lb, and 48lb at 6d. per lb, and 24lb at 4d. per lb, to make the said Quantity, and to bear the said Price.

And if you had worked as the Branches are linked from the third Order, then you would have found the Quantity of 36lb of each.

*Quest. 3.* A Vintner hath 4 Sorts of Wine, viz. Canary at 10s. per Gallon, Malaga at 8s. per Gallon, Rhenish Wine at 6s. per Gallon, and White Wine at 4s. per Gallon, and he is minded to make a Composition of them all of 60 Gallons, that they may be worth 5s. per Gallon, I desire to know how much of each he must have?

The Number of Terms being ranked according to the second Rule of this Chapter, the Branches will be linked as followeth, but will admit of no other manner of coupling, because there is but one Branch that is lesser than the Root, therefore all the rest must be linked unto it; and the Difference between the

Root and the three first Branches, viz. 10, 8 and 6, which are 5, and 3 and 1 must be set against 4 because they are coupled with it; and the Difference between the Root, viz. 5 and 4, which

is 1, must be set against the three other, because it is linked to them all; so I find 1 Gallon of Canary, 1

10	1	1
8	1	1
6	1	1
4	5, 3, 1	9



Gallon of Malaga, 1 Gallon of Rhenish Wine, and 9 Gallons of White Wine, priced as above, being mingled together, will be worth 5s. per Gallon, the Sum being 12 Gallons; but there must be 60 Gallons, wherefore I say,

As 12 is to 1, so is 60 to 5 Gallons of Canary.

As 12 is to 1, so is 60 to 5 Gallons of Malaga.

As 12 is to 1, so is 60 to 5 Gallons of Rhenish.

As 12 is to 9, so is 60 to 45 Gallons of White Wine. so that 5 Gallons of Canary, 5 Gallons of Malaga 5 Gallons of Rhenish, and 45 of White Wine mingled together, will be in all 60 Gallons worth 5s. per Gallon, which was required.

*Quest. 4.* A Goldsmith hath Gold of four several Sorts of Fineness, viz. of 24 Carects fine, and of 22 Carects fine, and of 20 Carects fine, and of 15 Carects fine, (*Read Ch p. 2. Def. of this Book*) and he would mingle so much of each with Alloy, that the whole Mass of 28oz of Gold so mingled may bear 17 Carects fine; I demand how much of each he must take? The 2d and 3d Rules of this Chapter being observed, (for instead of the Alloy I put 0, because it bears no Fineness, but it makes a Branch in the Operation) the Terms may be alligated, and the Differences added by any of these four Ways following, viz.

First thus,

17	{	24		17		17
		22		2		2
		20		2, 17		19
		15		5, 3		8
		0		7, 3		10
						Sum 56

Secondly thus,

17	{	24		2		2
		22		17		17
		20		2, 17		19
		15		7, 3		10
		0		5, 3		8
						Sum 56

Thirdly

Thirdly thus,

17	24	3	2
	22	2	2
	20	2, 17	19
	15	7, 5, 3	15
	0	3	3

Sum 14.

Fourthly thus,

17	0	7, 5, 3	15
	15	7, 5, 3	15
	20	2, 17	19
	22	2, 17	19
	24	2, 17	19

Sum 87

More Ways may be given for the alligating or linking of the Terms is this Question, but these, if well practised, are sufficient for understanding the Rules of Alligation.

In Questions of Alligation Total the Answer is given true, when the Sum of each of the Quantities of Simples found, agrees with the Sum or Quantity propounded; as in the last Question, the Answer was 8oz. 10 pwt. of

24 Carects fine, 1oz. of 22 Carects fine, 9oz. 10 pwt. of 20 Carects fine, 4oz. of 15 Carects fine, and 5oz. of Alloy, which added together make 28oz. the Quantity propounded.

## CHAP. XIX.

### Reduction of Vulgar Fractions.

1. **W**HAT a Vulgar Fraction is, hath been already shewed in the 1st Chapter of this Book, to which refer the Reader to look cautiously into.

2. To reduce a Vulgar Fraction, observe carefully these eight following Rules,

1. To reduce a mix'd Number into an improper Fraction.

2. To reduce a whole Number into an improper Fraction.

3. To reduce an improper Fraction into its equivalent whole (or mix'd) Number.

4. To

4. To reduce a Fraction into the lowest Terms equivalent to the Fraction given.

5. To find the Value of a Fraction in the known Parts of Coin, Weight, Measure, &c.

6. To reduce a compound Fraction to a simple one of the same Value.

7. To reduce diverse Fractions having unequal Denominations, to Fractions of the same Value having an unequal Denominator.

8. To reduce a Fraction of one Denomination to another of the same Value.

I. To reduce a mix'd Number to an improper Fraction.

The Rule is,

Multiply the Integer Part (or whole Number) by the Denominator of the Fraction, (*Vide Chap. 1. Defin. 31.*) and to the Product add the Numerator, and that Sum place over the Denominator for a new Numerator, so this new Fraction shall be equal to the mix'd Number given. As for Example :

1. Reduce  $18\frac{3}{7}$  into an improper Fraction ; multiply the whole Number 18 by 7 the Denominator, and to the Product add the Numerator 3, the Sum is 129, which put over the Denominator 7, and it makes  $18\frac{3}{7}$  for the Answer, as followeth.

$$\begin{array}{r} 18\frac{3}{7} \\ \hline 7 \\ \hline \text{Facit } 129 \\ \hline 7 \end{array}$$

2. Reduce  $183\frac{4}{11}$  to an improper Fraction, *facit*,  $2002\frac{4}{11}$ .

3. Reduce  $50\frac{1}{12}$  to an improper Fraction, *facit*,  $601\frac{1}{12}$ .

II. To reduce a whole Number into an improper Fraction.

The Rule is, Multiply the given Number by the intended Denominator and place the Product for the Numerator over it. (*Vide Chap. 1. Defin. 23.*) As for Example :

1. Let it be required to reduce 15 into a Fraction whose Denominator shall be 12. To effect

which, I multiply 15 by the intended Denominator (12) the Product is 180 which I place over 12 as a Numerator, and it makes  $15\frac{0}{12}$ , which is equal to 15 as was required ; as per Margent.

$$\begin{array}{r} 15 \\ \times 12 \\ \hline 30 \\ \times 15 \\ \hline \text{Facit } 180 \end{array}$$

2. Reduce

2. Reduce  $\frac{36}{26}$  into an improper Fraction, whose Denominator shall be 26, *facit*  $9\frac{36}{26}$ .

3. Reduce  $\frac{135}{16}$  into an improper Fraction, whose Denominator shall be 16, *facit*  $8\frac{15}{16}$ .

III. To reduce an improper Fraction into its equivalent whole or mix'd Number.

The Rule is, Divide the Numerator by the Denominator, and the Quotient is the whole Number equal to the Fraction; and if any Thing remain, put it for a Numerator over the Divisor.

Example.

1. Reduce  $43\frac{6}{8}$  into its equivalent mix'd Number. Divide the Numerator 436 by the Denominator 8, and the Quotient is 54, and 4 remains, which put for a Numerator over the Divisor 8, the Answer is  $54\frac{4}{8}$ , as followeth.

8) 436 (54

$$\begin{array}{r} 40 \\ 36 \\ 32 \\ \hline (4) \end{array}$$

Facit  $54\frac{4}{8}$

2. Reduce  $24\frac{76}{15}$  to a mix'd Number. *Facit*  $231\frac{11}{15}$ .

3. Reduce  $15\frac{876}{138}$  to a mix'd Number. *Facit*  $114\frac{72}{138}$ .

IV. To reduce a Fraction into its lowest Terms, equivalent to the Fraction given.

The Rule is, 1. If the Numerator and Denominator are even Numbers, take half the one and half of the other, as often as may be, and when either of them falls out to be an odd Number, then divide them by any Number that you can discover will divide both Numerator and Denominator without any Remainder; and when you have thus proceeded as low as you can reduce them, then this new Fraction so found out shall be the Fraction you desire, and will be equal in Value to the given Fraction.

Example 1. Let it be required to reduce  $\frac{192}{336}$  into its lowest Terms. First I take the half of the Numerator 192, and it is 96, then half of the Denominator, and it is 168, so that it is brought to  $\frac{96}{168}$ , and next to  $\frac{48}{84}$ , and by halving still to  $\frac{24}{42}$ , and their half is  $\frac{12}{21}$ , and now I can no longer half it, because 21 is an odd &c. and I find 3 divides them both without any Remainder, and brings them to  $\frac{4}{7}$ , as per Margent.

F 4

So

So I conclude  $\frac{1}{3}$  thus found to be equal in Value to the given Fraction  $\frac{192}{336}$ .

2. What is  $\frac{1}{16}$  in its lowest Terms? *Answer*,  $\frac{1}{16}$ .

3. What is  $\frac{1342}{1588}$  in its lowest Terms? *Answer*,  $\frac{11}{13}$ .

The best way to reduce a Fraction into its lowest Terms is, by finding a common Measure, viz. the greatest Number that will divide the Numerator and Denominator without any Remainder, and by that means reduce a Fraction to its lowest Terms at the first Work; and to find out this common Measure, divide the Denominator by the Numerator, and if any Thing remain divide your Divisor thereby, and if any Thing yet remain, then divide your last Divisor by it; do so until you find nothing remaining, then this last Divisor shall be your greatest common Measure, which will divide both Numerator and Denominator, and reduce them both into their lowest Terms at one Work.

*Example 4.* Reduce  $\frac{228}{304}$  into its lowest Terms by a common Measure; to effect which I divide the Denominator 304 by the Numerator 228, and there remains 76; then I divide 228 (the first Divisor) by 76 (the Remainder) and it quotes 3, and nothing remains; wherefore the last Divisor 76 is the common Measure, by which I divide the Numerator of the given Fraction; viz. 228, it quotes 3 for a new Numerator; then I divide the Denominator 304 by 76, and it quotes 4 for a new Denominator, so that now I have found  $\frac{3}{4}$  equal to  $\frac{228}{304}$ .

5. Reduce  $\frac{5048}{7392}$  into its lowest Terms by a common Measure, *facit*  $\frac{2}{3}$ .

6. Reduce  $\frac{3081}{20382}$  into its lowest Terms by a common Measure, *facit*  $\frac{1}{66}$ .

#### *A Compendium.*

*Note*, That if the Numerator and Denominator of a Fraction end each with a Cypher or Cyphers, then cut off as many Cyphers from the one as from the other, and the remaining Figures will be a Fraction of the same Value, viz.  $\frac{3400}{7100}$  will be found to be reduced to  $\frac{34}{71}$ , by cutting off the two Cyphers from the Numerator and Denominator with a Dash of the Pen, thus,  $\frac{34}{71}$ , and  $\frac{400}{700}$  will be  $\frac{4}{7}$ , thus,  $\frac{400}{700}$ , &c.

V. To find the Value of a Fraction in the known Parts of Coins, Weights, &c.

The Rule is, Multiply the Numerator by the Parts of the next inferior Denomination that are equal to an Unit of the



the same Denomination with the Fraction ; then divide the Product by that Denominator, and the Quote gives you its Value in the same Parts you multiplied by, and if any Thing remain, multiply it by the Parts of the next inferior Denomination, and divide as before ; do so till you can bring it no lower, and the several Quotients will give you the Value of the Fraction as was required ; and if any at last remain, place it for a Numerator over the former Denominator. Some few Examples will make the Rule plain.

1. What is the Value of  $\frac{27}{29}$  l. sterling ? To answer this Question, I multiply the Numerator 27 by 20, (the Shillings in a Pound) the Product is 540, which I divide by 29 (the Denominator) and the Quotient is 18s. and there remains 18, which I multiply by 12 Pence, and the Product (216) I divide by the Denominator 29, the Quotient is 7d. and 13 remains, which I multiply by 4 Farthings, the Product is 52, which I still divide by 29, the Quotient is 1qr. and there remaineth 23, which I put for a Numerator over the Denominator 29, to find the Value of  $\frac{23}{29}$  to be 18s. 7d. 1qr.  $\frac{23}{29}$ , as by the Work in the Margent, and after the same manner the Value of  $\frac{1}{12}$  of a Pound sterling is found out to be 4s. 8d.

$$\begin{array}{r} \frac{27}{29} \text{ l.} \\ \text{Multiply } 20 \\ 29) 540 \text{ (18s. 7d. } 1\frac{23}{29}\text{q.} \end{array}$$

$$\begin{array}{r} 29 \\ \hline 250 \\ 232 \\ \hline \text{Rem. (18)} \\ \text{Mult. } 12 \\ \hline 36 \\ 18 \\ \hline 29) 216 \text{ (7d.} \\ 203 \end{array}$$

$$\begin{array}{r} \text{Rem. (13)} \\ \text{Mul. } 4 \\ \hline \text{--- gr.} \\ 29) 52 \text{ (1}\frac{23}{29}\text{qr.} \\ 29 \\ \hline \text{Rem. (23)} \end{array}$$

$$\begin{array}{r} \text{s. d. qr.} \\ \text{Facit } 18 \quad 7 \quad 1\frac{23}{29} \end{array}$$

And so likewise you may find the Value of any Fraction either in Weight or Time, &c.

To reduce a compound Fraction to a Simple of the same Value.

What a compound Fraction is, hath been shewn in Chap Definition 24, and to reduce it to a simple Fraction of the same Value.

The Rule is ; Multiply the Numerators continually, and place the last Product for a new Numerator, then multiply the

the Denominators continually, and place the last Product for a new Denominator; so this single Fraction shall be equal to the compound Fraction.

*Example.*

1. Reduce  $\frac{2}{3}$  of  $\frac{3}{5}$  of  $\frac{5}{8}$  to a simple Fraction.

Multiplying the Numerators, 2, 3 and 5 together, they make 30 for a new Numerator; then I multiply the Denominators 3, 5 and 8 together, and their Product is 120 for a Denominator, so the simple Fraction is  $\frac{30}{120}$ , and cutting off the Cyphers it is  $\frac{1}{4}$ , equal to the  $\frac{1}{4}$  by the 4th Rule following.

$$\begin{array}{r} 5 \\ \times 3 \\ \hline 15 \\ \times 8 \\ \hline 120 \end{array}$$

$$\begin{array}{r} 3 \\ \times 2 \\ \hline 6 \\ \times 5 \\ \hline 30 \end{array}$$

*Facit*  $\frac{30}{120}$ , or  $\frac{1}{4}$ , or  $\frac{1}{4}$ .

2. What is  $\frac{2}{3}$  of  $\frac{5}{8}$  of  $\frac{4}{7}$  of  $\frac{11}{14}$ ? *Ans.*  $\frac{1140}{7350}$ , or  $\frac{114}{735}$ , or  $\frac{38}{245}$  in its lowest Terms.

3. What is  $\frac{11}{12}$  of  $\frac{13}{14}$  of  $\frac{21}{20}$ ? *Answer*,  $\frac{3003}{4480}$ .

By this you may know how to find the Value of a compound Fraction, viz. first reduce it to a simple one, and then find out its Value by the 5th Rule foregoing.

*Example 4.* What is the Value of  $\frac{3}{4}$  of  $\frac{5}{8}$  of  $\frac{9}{16}$  of Pound sterl.? *Ans.* 11s. 3d.

VII. To reduce Fractions of unequal Denominations to Fractions of the same Value, having equal Denominators.

- The Rule is, Multiply all the Denominators together, and the Product shall be the common Denominator; then multiply each Numerator into all the Denominators, except its own, and the last Product put for a Numerator over the Denominator found out as before; so this new Fraction is equal to that Fraction whose Numerator you multiplied into the said Denominator. Do so by all the Numerators given, and you have your Desire.

*Example.*

1. Reduce  $\frac{3}{4}$ ,  $\frac{4}{5}$ ,  $\frac{5}{6}$  and  $\frac{7}{8}$  to a common Denominator. Multiply the Denominators 4, 5, 6 and 8 together continually, and the Product is 960 for the common Denominator; then multiply the Numerator 3 into the Denominators, 5, 6 and 8, and the Product is 720, which is the Numerator to 960 (found as before) so  $\frac{720}{960}$  is equal to

first Fraction  $\frac{3}{4}$ ; then I proceed to find a new Numerator to the second Fraction, viz.  $\frac{4}{5}$ , and I multiply 4 (into all the Denominators except its own, viz.) into 4, 6 and 8, which produceth  $\frac{768}{960}$ , equal to  $\frac{4}{5}$ , then multiply the Numerator 5 into the Denominators, 4, 5 and 8, the Product is  $\frac{800}{960}$ , equal to  $\frac{5}{8}$ , then multiply the Numerator 7 into the Denominators 4, 5 and 6, the Product is  $\frac{840}{960}$ , equal to  $\frac{7}{6}$  and the Work is done: So that for  $\frac{3}{4}$ ,  $\frac{4}{5}$  and  $\frac{7}{6}$  I have  $\frac{768}{960}$ ,  $\frac{800}{960}$  and  $\frac{840}{960}$ .

2. Reduce  $\frac{11}{12}$ ,  $\frac{14}{15}$  and  $\frac{19}{21}$  to a common Denominator, *Ans.*  $\frac{5313}{3795}$ ,  $\frac{3528}{3795}$  and  $\frac{2444}{3795}$ .

VIII. *To reduce a Fraction of one Denomination to another.*

1. This is either ascending or descending. Ascending, when a Fraction of a smaller is brought to a greater Denomination. Descending when a Fraction of a greater Denomination is brought lower.

2. When a Fraction is to be brought from a lesser to a greater Denomination, then make of it a compound Fraction, by comparing it with the intermediate Denominations between it, and that you would have it reduced to; then (by the 6th Rule foregoing) reduce your Compound to a simple Fraction, and the Work is done.

*Example.*

*Quest.* 1. It is requir'd to know what Part of a Pound Sterling  $\frac{5}{8}$  of a Penny is?

To resolve this, I consider that 1d. is  $\frac{1}{20}$  of a Shilling, and a Shilling is  $\frac{1}{20}$  of a Pound; wherefore  $\frac{1}{20}$  is  $\frac{1}{400}$  of a Pound, which, by the said 9th Rule, I find to be  $\frac{5}{1600}$  of a Pound *Sterl.* of *English* Money.

*Quest.* 2. What Part of a Pound *Troy-weight* is  $\frac{1}{2}$  of a penny-weight? *Ans.*  $\frac{1}{2}$  of  $\frac{1}{20}$  of  $\frac{1}{12}$ , equal to  $\frac{1}{480}$  *Troy*.

3. When a Fraction is to be brought from a greater to a lesser Denomination, then multiply the Numerator by the parts contained in the several Denominations betwixt it and the Parts you would reduce it to; then place the last product over the Denominator of the given Fraction.

*Example.*

*Quest.* 3. I would reduce  $\frac{3}{4}$  *l.* to the Fraction of 1d. to which, I multiply the Numerator 3 by 20 and 12, the product is 720, which I put over the Denominator 5, it makes  $\frac{720}{5}$  of 1d. equal to  $\frac{144}{1}$ .

*Quest.* 4. What Part of an Ounce *Troy* is  $\frac{1}{16}$  lb? *Answer*  $\frac{1}{16}$ .

## CHAP. XX.

*Addition of Vulgar Fractions.*

1. IF your Fractions to be added have a common Denominator, then add all the Numerators together, and place their Sum for a Numerator to the common Denominator, which new Fraction is the Sum of all the given Fractions; and if it be improper, reduce it to a whole or mix'd Number, by the 3d Rule in the 19th Chapter.

*Quest. 1.* What is the Sum of  $\frac{7}{24}$ ,  $\frac{9}{24}$ ,  $\frac{16}{24}$  and  $\frac{14}{24}$ ?

The Denominators are equal, viz. every one is 24; wherefore add the Numerators together, viz. 7, 9, 16 and 14, their Sum is 46, which put over the Denominator 24 it makes  $\frac{46}{24}$ , the Sum of the given Fractions, which will be reduced to the mixed Numbers  $1\frac{23}{12}$  or  $1\frac{11}{6}$ .

2. But if the Fractions to be added have unequal Denominators, then reduce them to a common Denominator by the 7th Rule of Chap. 19, and then add the Numerators together, and put the Sum over the common Denominator &c. as before in the last Example.

*Quest. 2.* What is the Sum of  $\frac{3}{4}$ ,  $\frac{7}{8}$ ,  $\frac{9}{10}$  and  $\frac{11}{12}$ ?

The Fractions reduced to a common Denominator are  $\frac{900}{2400}$ ,  $\frac{2800}{2400}$ ,  $\frac{2160}{2400}$  and  $\frac{2200}{2400}$ , the Sum of their Numerators is 15800, which put over the common Denominator makes  $\frac{15800}{2400}$ , or  $6\frac{145}{60}$ , equal to the mixed Number  $3\frac{1}{4}$  for the Sum required.

*Quest. 3.* What is the Sum of  $\frac{18}{11}$ ,  $\frac{21}{19}$  and  $\frac{36}{17}$ ?

*Answer,*  $1\frac{37525}{39151}$ .

3. If you are to add mixed Numbers together, then add the fractional Parts as before, and if their Sum be an improper Fraction, reduce it to a mixed Number, and add its integral Part to the integral Parts of the given mixed Numbers, and the Work is done.

*Quest. 4.* What is the Sum of  $13\frac{3}{4}$  and  $21\frac{1}{5}$ ?

First add the Fractions  $\frac{3}{4}$  and  $\frac{1}{5}$ : the Sum is  $1\frac{13}{20}$ , then add the Integer 1 to 13 and 21, their Sum is 38, and put after the Fraction  $\frac{13}{20}$ , it is  $38\frac{13}{20}$  for the Answer, or it is  $38\frac{1}{2}$ .

*Quest. 5.* What is the Sum of  $48\frac{3}{4}$ ,  $64\frac{1}{2}$  and  $130\frac{1}{4}$ ?

*Facit*  $243\frac{180}{220}$ , or  $243\frac{9}{11}$ .

4. If any of the Fractions to be added is a compound Fraction, it must first be reduced to a simple Fraction by the 6th Rule of Chapter 19, and then add it to the rest, according to the 2d Rule of this Chapter.

*Example.*

*Quest. 6.* What is the Sum of  $\frac{3}{4}$ ,  $\frac{5}{6}$  and  $\frac{7}{8}$  of  $\frac{3}{4}$  of  $\frac{5}{6}$ ?  
Reduce  $\frac{7}{8}$  of  $\frac{3}{4}$  of  $\frac{5}{6}$  into a simple Fraction, and it is  $\frac{105}{192}$ , which reduced with the other two, and added, are  $24\frac{600}{800}$ .

*Quest. 7.* What is the Sum of  $1\frac{1}{2}$  and  $\frac{3}{4}$  of  $\frac{4}{5}$  of  $\frac{5}{6}$ ?

*Answer.*  $1\frac{5}{12}$ .

5. If the Fractions to be added are not of one Denomination, they must be so reduced, and then proceed as before.

*Quest. 8.* What is the Sum of  $\frac{3}{4}$  *l.* and  $\frac{5}{6}$  *s.*?

Of the given Fractions here, one is of a Pound, and the other the Fraction of a Shilling; and before you can add them together you must reduce  $\frac{5}{6}$  *s.* to the Fraction of a Pound as the other is (by the 8th Rule of Chap. 19.) and it makes  $17\frac{5}{20}$  *l.* then  $\frac{3}{4}$  and  $17\frac{5}{20}$  *l.* will be found to be  $18\frac{15}{20}$  *l.* or  $18\frac{3}{4}$  *l.* by the 7th Rule of Chap. 19. and in its lowest Terms  $18\frac{3}{4}$  *l.* by the 4th Rule of Chap. 19.

It would have been the same (if by the latter Part of the 8th Rule of Chapter 19) you had reduced  $\frac{3}{4}$  *l.* to the Fraction of a Shilling, which you would have found to have been  $6\frac{3}{4}$  *s.* which added to  $\frac{5}{6}$  *s.* by the said 17th Rule of the last Chapter, the Sum is  $15\frac{20}{24}$  *s.* which is equal to the Sum found, as before, viz.  $18\frac{3}{4}$  *l.* for (by the 5th Rule of Chapter 19.) the Value of  $18\frac{3}{4}$  *l.* will be found to be  $15\frac{10}{12}$  *s.* and so will  $15\frac{20}{24}$  *s.* be found to be just as much.

*Quest. 9.* What is the Sum of  $\frac{2}{3}$  *l.*  $\frac{3}{4}$  *s.* and  $\frac{3}{4}$  *d.*?

*Ans.*  $1\frac{525}{1200}$  or  $1\frac{21}{48}$  *l.* or in its lowest Terms  $1\frac{7}{16}$ .

## CH A P. XXI

### *Subtraction of Vulgar Fractions.*

THE Rules in Addition for reducing the given Fractions to one Denomination, are here to be observed; for before Subtraction can be made, the Fractions must be reduced to a common Denominator; then subtract the Numerator from the other, and place the Remainder over a common Denominator, which Fraction shall be the Excess or Difference between the given Fractions.

*Example.*

*Quest. 1.* What is the Difference between  $\frac{3}{4}$  and  $\frac{5}{6}$ ? The given Fractions are reduced to  $\frac{9}{12}$  and  $\frac{10}{12}$ , then subtract



the Numerator 20 from the Numerator 21, and there remains 1, which being put over the Denominator 28 makes  $\frac{1}{28}$  for the Answer or Difference between  $\frac{3}{4}$  and  $\frac{5}{8}$ .

*Quest. 2.* What is the Difference between  $\frac{5}{8}$  and  $\frac{3}{4}$  of  $\frac{5}{8}$ ?  
Reduce the compound Fraction  $\frac{3}{4}$  of  $\frac{5}{8}$  to a simple Fraction, then proceed as before, and the Answer is  $\frac{11}{240}$ , equal to  $\frac{11}{240}$ .

2. When a Fraction is given to be subtracted from a whole Number, subtract the Numerator from the Denominator, and put the Remainder for a Numerator to the given Denominator, and subtract an Unit (for that you borrowed) for the whole Number, and the Remainder place before the Fraction found, as before, which mixed Number is the Remainder or Difference sought.

*Example.*

*Quest. 3.* Subtract  $\frac{7}{10}$  from 48.

*Answer,* 47  $\frac{3}{10}$ , for if you subtract 7 (the Numerator) from 10 (the Denominator) there remains 3, which put over 10 is  $\frac{3}{10}$ , and 1 (1 borrowed) from 48 rests 47, to which join  $\frac{3}{10}$ , and it makes 47  $\frac{3}{10}$  for the Excess.

*Quest. 4.* Subtract  $\frac{13}{18}$  from 57, remain 56  $\frac{5}{18}$ .

3 If it be required to subtract a Fraction from a mixed Number, or one mixed Number from another, reduce the Fraction to a common Denominator, and if the Fraction to be subtracted be lesser than the other, then subtract the lesser Numerator from the greater, and that is a Numerator for the common Denominator; then subtract the lesser integral Part from the greater, and the Remainder, with the remaining fractions thereunto annexed, is the Difference required between the two given mixed Numbers.

*Quest. 5.* Subtract 26  $\frac{3}{4}$  from 54  $\frac{5}{8}$ .

First, subtract  $\frac{3}{4}$ , viz.  $\frac{12}{16}$  from  $\frac{5}{8}$ , viz.  $\frac{10}{16}$ , the Remainder is  $\frac{17}{16}$ ; then 26 from 54 remaineth 28, to which annex  $\frac{17}{16}$ , it makes 28  $\frac{17}{16}$  for the Answer.

4. But if the Fraction to be subtracted is greater than the Fraction from whence you subtract, then having first reduced the Fractions to a common Denominator, take the Numerator of the greatest Fraction out of the Denominator, and add the Remainder to the Numerator of the lesser Fraction, and their Sum is a new Numerator to the common Denominator, which Fraction note; then (for the 1 you borrowed) add 1 to the integral Part to be subtracted, and subtract it from the greater Number, and to the Remainder annex the Fraction you noted before, to this new mixed Number shall be the Difference sought.

*Example.*

*Example.**Quest. 6.* Subtract  $14\frac{3}{4}$  from  $29\frac{1}{4}$ .

The Fractions reduced are, viz.  $\frac{3}{4}$  equal to  $\frac{21}{28}$ , and  $\frac{1}{4}$  equal to  $\frac{7}{28}$ ; now I sh<sup>d</sup> subtract  $\frac{21}{28}$  from  $\frac{7}{28}$ , but I cannot, therefore I subtract 21 from 28, rest 7, which added to 16 (the lesser Numerator) make 23 for a Numerator to 28, viz.  $\frac{23}{28}$ ; then I come to the integral Parts 14 and 29, and say, 1 that I borrowed and 14 is 15, which taken from 29 there rests 14, to which annexing  $\frac{23}{28}$ , it is  $14\frac{23}{28}$ , for the Remainder or Difference between  $14\frac{3}{4}$  and  $29\frac{1}{4}$ .

*Quest. 7.* Subtract  $36\frac{1}{10}$  from  $74\frac{1}{2}$ . *Facit*  $37\frac{9}{10}$ .

## C H A P. XXII.

*Multiplication of Vulgar Fractions:-*

1. IF the Multiplicand and Multiplier are simple Fractions I then multiply the Numerators together for a new Numerator, and the Denominators for a new Denominator, and the new Fraction is the Product required.

*Quest. 1.* What is the Product of  $\frac{5}{7}$  by  $\frac{9}{11}$ ? *Facit*  $\frac{45}{77}$ ; for the Numerators 5 and 9 being multiplied make 45, and the Denominators 7 and 11 being multiplied make 77.

*Quest. 2.* What is the Product of  $\frac{11}{12}$  by  $\frac{21}{32}$ ? *Facit*  $\frac{231}{384}$ .

2. If the Fractions be multiplied by mix'd Numbers, reduce them to improper Fractions by the first Rule of the 19th Chapter, then proceed as before.

*Quest. 3.* What is the Product of  $48\frac{3}{5}$  by  $13\frac{5}{8}$ ?

The given mix'd Numbers being reduced to improper Fractions are  $48\frac{3}{5}$  equal to  $24\frac{3}{5}$ , and  $13\frac{5}{8}$  equal to  $10\frac{5}{8}$ ; now  $24\frac{3}{5}$  multiplied by  $10\frac{5}{8}$ , according to the 1st Rule of this Chapter, produceth  $201\frac{15}{40}$ , or  $672\frac{3}{8}$ .

*Quest. 4.* What is the Product of  $439\frac{6}{10}$  by  $18\frac{3}{4}$ ? *Facit*  $6708\frac{4}{5}$  or  $8104\frac{1}{2}$ .

3. If a compound Fraction is to be multiplied by a simple Fraction, first reduce the compound Fraction into a simple Fraction, then multiply the one by the other, as is taught above.

*Quest. 5.* What is the Product of  $\frac{16}{21}$  by  $\frac{3}{4}$  of  $\frac{5}{7}$  of  $\frac{4}{5}$ ?

The Compound Fraction  $\frac{3}{4}$  of  $\frac{5}{7}$  of  $\frac{4}{5}$  reduced is  $\frac{16}{140}$  or  $\frac{2}{17.5}$ , which multiplied by  $\frac{16}{21}$  produceth  $\frac{32}{357}$ , which in its lowest Terms is  $\frac{16}{178.5}$  for the Answer.

And if the Multiplicand and Multiplier are both compound Fractions, reduce them both to simple ones, then multiply these few Fractions as before, so you have the Product.

*Quest. 6.* What is the Product of  $\frac{3}{4}$  of  $\frac{2}{3}$  of  $\frac{3}{5}$  by  $\frac{1}{2}$ ?

*Answer*  $\frac{18}{120}$ , in its lowest Term  $\frac{3}{20}$ .

*Quest. 7.* What is the Product of  $\frac{2}{3}$  of  $\frac{3}{4}$  by  $\frac{3}{5}$  of  $\frac{4}{6}$ ?

*Answer*  $\frac{60}{360}$ , or  $\frac{1}{6}$ , or in its least Term  $\frac{1}{6}$ .

4. If a Fraction be to be multiplied by a whole Number, put under the given whole Number an Unit for a Denominator, whereby it will be an improper Fraction, then multiply the Fractions as before.

#### Example.

*Quest. 8.* What is the Product of 24 by  $\frac{2}{3}$ ?

*Answer*  $4\frac{8}{3}$ ; for 24 by putting an Unit under it will be  $24\frac{1}{1}$ , and  $24\frac{1}{1}$  by  $\frac{2}{3}$  produceth  $4\frac{8}{3}$  or 16.

*Quest. 9.* What is the Product of 36 by  $\frac{2}{11}$ ?

*Answer*  $3\frac{24}{11}$ , or  $29\frac{5}{11}$ .

## C H A P. XXIII.

### Division of Vulgar Fractions.

1. IF the Dividend and Divisor are both simple Fractions, then multiply the Numerator of the Dividend into the Denominator of the Divisor, and the Product is a new Numerator, and multiply the Denominator of the Dividend into the Numerator of the Divisor, and the Product is a new Denominator, which new Fraction thus found is the Quotient you desire.

#### Example.

*Quest. 1.* What is the Quotient of  $\frac{5}{8}$  divided by  $\frac{3}{5}$ ?

*Ans.*  $\frac{25}{24}$ , or  $1\frac{1}{24}$ ; for first I multiply (5) the Numerator of the Dividend into (5) the Denominator of the Divisor, and the Product (25) is a Numerator for the Quotient, then I multiply (8) the Denominator of the Dividend into (3) the Numerator of the Divisor, and the Product (24) I put in the Quotient for a Denominator; so I find  $\frac{25}{24}$  is the Quotient sought.

$$\begin{array}{r} 3 \quad 5 \quad 25 \\ 5 \quad 8 \quad 24 \end{array}$$

*Quest. 2.* What is the Quotient of  $\frac{10}{21}$  divided by  $\frac{2}{3}$ ?

*Answer*  $\frac{30}{42}$ , equal to  $\frac{5}{7}$  in its lowest Terms.

2. But if you will divide a simple Fraction by a Compound, or a Compound by a simple, first reduce such Compound to a simple Fraction, then go on as before.

*Quest. 3.* What is the Quotient of  $\frac{3}{10}$  divided by  $\frac{3}{4}$  of  $\frac{3}{4}$ ?

*Answer*  $\frac{36}{80}$  or  $\frac{9}{20}$ ; first reduce  $\frac{3}{4}$  of  $\frac{3}{4}$  into a simple Fraction, and it is  $\frac{9}{16}$ , by which  $\frac{3}{10}$  being divided, the Quotient is  $\frac{9}{160}$ , equal in its least Terms to  $\frac{9}{160}$ ; and if the Dividend and Divisor be both of compound Fractions, reduce them both to a simple Fraction, then divide the one by the other, as in Rule 1. foregoing.

*Quest. 4.* What is the Quote of  $\frac{2}{3}$  of  $\frac{3}{4}$  divided by  $\frac{2}{5}$  of  $\frac{5}{6}$ ?

*Answer*  $\frac{180}{120}$  or  $\frac{3}{2}$ , or  $1\frac{1}{2}$  in its lowest Terms.

3. If the Dividend, or Divisor, or both, are mixed Numbers, reduce them to improper Fractions, and perform Division as you are taught before.

*Quest. 5.* What is the Quote of  $12\frac{3}{4}$  divided by  $21\frac{4}{5}$ ?

*Answer*  $\frac{255}{438}$  for  $12\frac{3}{4}$ , is equal to  $5\frac{1}{4}$ , and  $21\frac{4}{5}$  is equal to  $4\frac{1}{5}$ , and the Quote of  $5\frac{1}{4}$  divided by  $4\frac{1}{5}$  is as before  $\frac{255}{438}$ .

4. If you divide a Fraction by a whole Number, or a whole Number by a Fraction, make the whole Number an improper Fraction, by putting an Unit for a Denominator to it, as was taught in Rule 4. Chap. 22. and then perform Division as was before taught.

#### Example

*Quest. 6.* What is the Quote of 8 divided by  $\frac{3}{5}$ ?

*Answer*  $4\frac{2}{3}$ , which is equal to  $13\frac{1}{3}$ . 
$$\begin{array}{r} 3 \overline{) 8} \\ \underline{40} \text{ or } 13\frac{1}{3} \end{array}$$

See the Work in the Margent.

*Quest. 7.* What is the Quotient of  $\frac{3}{5}$  divided by  $\frac{8}{1}$ ?

*Answer*  $\frac{3}{40}$ , as per Margent.

$$\begin{array}{r} 8 \overline{) 3} \\ \underline{1} \quad 5 \quad 40 \end{array}$$

## C H A P. XXIV.

### be Rule of Three Direct in Vulgar Fractions.

AS in the Rule of Three in whole Numbers, so likewise in Fractions, you must see that the Fractions of the first and third Places be of the same Denomination.

1. If any of the given Fractions be compound, let 'em be reduced to simple of the same Value.

2. If there are given mixed Numbers, reduce them to proper Fractions by the first Rule of Chap. 19.





In resolving the last Question and the two next, observe the 3d Rule of the Chapter foregoing.

Quest. 5. If  $\frac{2}{10}$  of a C. cost 284s. what will  $7\frac{1}{2}$  C. cost at that Rate?

Answer 2368 $\frac{2}{3}$ s. or 118l. 6s. 8d.

Quest. 6. If  $3\frac{1}{4}$  Yards of Velvet cost 3 $\frac{5}{8}$ l. how much will 10 $\frac{1}{2}$  Yards cost at that Rate?

Answer 11 $\frac{3}{4}$ l.

Quest. 7. If 3 Yards of Broad-cloth cost 2 $\frac{1}{4}$ l. what will 14 $\frac{3}{4}$  Yards cost?

Answer 13l. 9s. 4d.

In working the last Question and the four next, observe the 4th Rule of the Chapter foregoing.

Quest. 8. If 14 $\frac{1}{2}$  lb of Pepper cost 14s. 6 $\frac{3}{4}$ d. I demand the Price of 73 $\frac{3}{4}$  lb?

Answer 3l. 16s. 7 $\frac{3}{8}$ d.

Quest. 9. If 1 $\frac{1}{2}$  lb of Cochineel cost 1l. 5s. what will 36 $\frac{2}{10}$  lb cost?

Answer 45l. 17s. 6d.

Quest. 10. If 1 Yard of Broad-cloath cost 15 $\frac{5}{8}$ s. what will 4 Pieces; each containing 27 $\frac{3}{4}$  Yards cost at that Rate?

Answer 85l. 14s. 3 $\frac{3}{4}$ d.

Quest. 11. A Mercer bought 3 $\frac{1}{2}$  Pieces of Silk, each Piece contained 24 $\frac{2}{3}$  Ells, at 6s. 0 $\frac{3}{4}$ d. per Ell, I demand the Value of 3 $\frac{1}{2}$  Pieces at that Rate?

Answer 26l. 3s. 3d.

In resolving the four next Questions, observe the 8th Rule of Chap. 19.

Quest. 12. If  $\frac{2}{3}$  of an Ounce of Silver cost 2s. I demand the Price of 11 $\frac{2}{3}$  at that Rate?

Answer 35l.

Quest. 13. If 5 $\frac{5}{8}$  lb of Gold is worth 205l. 14s. 3 $\frac{3}{4}$ d. Sterling, what is a Grain worth at that Rate?

Answer 1 $\frac{1}{2}$ d.

Quest. 14. If  $\frac{3}{4}$  Yards of Silk is worth  $\frac{3}{4}$  of  $\frac{1}{2}$ l. what is the Price of 15 $\frac{2}{3}$  Ells Flemish?

Answer 9l. 7s. 6d.

Quest. 15. If  $\frac{2}{3}$  of  $\frac{3}{4}$  of a Pound of Cloves cost 6s. 2 $\frac{1}{2}$ d. what cost the C. weight at that Rate?

Answer 69l. 6s. 8d.

Note, That when the Answers to the Questions in this and the next Chapter are given in Fractions, they are given in their lowest Terms.

## C H A P. XXV.

*The Rule of Three Inverse in Fractions.*

**I**T hath been already taught (in the 3<sup>d</sup> Rule of the 11<sup>th</sup> Chapter,) how to discover when the 4<sup>th</sup> Proportional Number (to the 3 given Numbers) is to be found out by a *Rule of Three Direct*, and when by a *Rule of Three Inverse*, to which Rule the Learner is now referred.

2. When (in Fractions) you find a Question to be resolved by the *Rule of Three Inverse*, viz. when the third Term is the Divisor, then having reduced the Terms exactly (according to the Rules in Chap. 24.) multiply the Numerators of the third Fraction into the Denominators of the second and first Fractions, and the Product is a new Denominator; then multiply the Denominator of the third Fraction into the Numerators of the second and first Fractions, and the Product is a new Numerator, which new Fraction thus found is the Answer to the Question.

*Quest. 1.* If  $\frac{3}{4}$  of a Yard of Cloth, that is two Yards wide, will make a Garment, how much of any other Drapery that is  $\frac{3}{5}$  of a Yard wide will make the same Garment?

*Answer.*  $2\frac{1}{2}$  Yards.

*Quest. 2.* I lent my Friend 46<sup>l</sup>. for  $\frac{4}{5}$  of a Year, how much ought he to lend me for  $\frac{7}{12}$  Parts of a Year?

*Answer*  $63\frac{31}{5}$  <sup>l</sup>.

*Quest. 3.* If  $\frac{2}{3}$  of a Yard of Cloth that is  $2\frac{1}{3}$  Yards wide will make any Garment, what Breadth is that Cloth when  $1\frac{1}{4}$  Yard will make the same Garment?

*Answer*  $\frac{56}{83}$  or  $\frac{8}{9}$  of a Yard wide?

*Quest. 4.* How many Inches in Length of a Board that is 9 Inches broad will make a Foot square?

*Answer* 16 Inches in Length.

*Quest. 5.* If when the Bushel of Wheat cost  $4\frac{1}{4}$  <sup>s</sup> the Penny Loaf weighed  $10\frac{2}{3}$  Ounces, what will it weigh when the Bushel cost  $8\frac{2}{10}$  <sup>s</sup>?

*A swer*  $5\frac{18\frac{1}{2}}{4}$  Ounces.

*Quest. 6.* If 17 Men can mow  $24\frac{1}{2}$  Acres in  $10\frac{2}{3}$  Days, in how many Days will 6 Men do the same? ..

*Answer* in  $21\frac{1}{3}$  Days.

C H A P.

## C H A P. XXVI.

*Rules of Practice.*

1. **I**N the single Rule of Three, when the first of the three Numbers in the Question (after they are disposed according to the 6th Rule of Chap. 10.) happeneth to be an Unit (or 1.) that Question many times may be resolved far more speedily than by the Rule of Three. which kind of Operation is commonly called *Practice*; and indeed it is of excellent Use among Merchants, Tradesmen and others, by reason of its speediness in finding a Resolution to such kind of Questions.

2. The chiefest Questions resolvable by these brief Rules, may be comprehended under the three general Heads or Cases following, *viz.*

- |   |   |   |
|---|---|---|
| When the<br>given Price<br>of the Inte-<br>ger consists | { | 1. Of Farthings under 4.                    |
|   |   | 2. Of Pence under 12.                       |
|   |   | 3. Of Pence and Farthings.                  |
|   |   | 4. Of Shillings under 20.                   |
|   |   | 5. Of Shillings, Pence and Farthings.       |
|   |   | 6. Of Pounds.                               |
|   |   | 7. Of Pounds, Shillings, Pence & Farthings. |

It would be very convenient for the practical Arithmetician to have by Heart the several Products of the nine Digits multiplied by 12, for his speedy reducing Pence into Shillings, and Shillings into Pence, which he may gain by the following Table.

	1		12
	2	} is {	24
	3		36
	4		48
12 Times	5		60
	6		72
	7		84
	8		96
	9		108

3. Shillings are practically reduced into Pounds thus, *viz.* cut off the Figure standing in the place of Units with a Dash of the Pen, and note it for Shillings, then draw a Line under the given Number, and take half the remaining figures (after the first is cut off, and set them under the Line, and they are so many Pounds; but if the last Figure is odd, then take the lesser half, and add 10 to the Figure

43658

l.

s.

2182

18

Figure so cutt off (as before) for Shillings; as if I were to reduce 43658 Shillings into Pounds, first I cut off the last Figure (8) for Shillings, then I take half of the remaining Figures (4365) thus, half of 4 is 2, which I put under the Line, then half of 3 is 1, and because 3 is an odd Number, I make the next Figure 6 to be 16, and I go on, saying, half of 16 is 8, then half of 5 is 2, which is the last Figure, wherefore, because 5 is an odd Number, I add 10 to the 8 I cut off, and it makes 18s. so that I find it to be 2182l. 18s. as per Margent.

4. It is likewise convenient that the Learner be acquainted with the practical Tables following, the first containing the aliquot or even Parts of a Shilling, the second containing the even Parts of a Pound.

The even Part of a Shilling.	6	}	is	}	The even Parts of a Pound.	10 00	}	is	}	l.
	4					6 08				
	3					5 00				
	2					4 00				
	1½					3 04				
	1					2 06				
						2 00				10 00
						1 08				10 00
						1 00				20 00

Case.

5. When the Price of an Integer is a Farthing; then take the 6th Part of the given Number, which will be so many Three-half-pences, and, if any thing remain it is Farthings, by the 7th Rule of Chap. 9. then consider, That Three-half-pence is  $\frac{1}{4}$  of a Shilling, wherefore take the 8th Part of them for Shillings: (and if any thing remain, they are so many Three-half-pences,) which reduce into Pounds by the 3d Rule foregoing.

Example.

What comes 67486lb to at a Farthing per lb? First, take  $\frac{1}{6}$  of 67486, and it is 11247 Three-half-pences, and 4 Farthings or 1 Penny; then  $\frac{1}{8}$  of 11247 is 1405s. and 7 remains, which is 7 Three-half-pences, or 10½d. which with the 4 Farthings before, make 11½d. and 1405s. which by the 3d Rule is 70l. 5s. in all 70l. 5s. 11½d. for the Answer. See the Work following.

$$\begin{array}{r|l}
 \frac{1}{2} & 67486 \text{ at } \frac{1}{4} \text{ per lb} \\
 \hline
 & \text{d.} \\
 \frac{1}{8} & 11247 \\
 \hline
 \frac{1}{10} & 1405 \\
 \hline
 & \text{l. s. d.} \\
 & 70 \quad 5 \quad 11\frac{1}{2} \text{ facit}
 \end{array}$$

Other Examples follow.

$$\begin{array}{r|l}
 \frac{1}{6} & 8576 \text{ lb at } 1 \text{ qr.} \\
 \hline
 \frac{1}{8} & 1429 \quad 2 \text{ qrs.} \\
 \hline
 \frac{1}{10} & 17 \quad 8 \quad 8 \text{ d.} \\
 \hline
 & \text{l. s. d.} \\
 & 8 \quad 18 \quad 8\frac{1}{2}
 \end{array}$$

$$\begin{array}{r|l}
 \frac{1}{6} & 6380 \text{ lb at } 1 \text{ qr.} \\
 \hline
 \frac{1}{8} & 1003 \quad 2 \text{ qrs.} \\
 \hline
 \frac{1}{10} & 12 \quad 2 \quad 11 \text{ d.} \\
 \hline
 & \text{l. s. d.} \\
 & 6 \quad 12 \quad 11\frac{1}{2} \text{ facit}
 \end{array}$$

6. When the Price of the Integer is two Farthings, then take the third Part of the given Number for so many Three-half-pences, and the Remainder, if any, is Half-pence, then take the eighth Part of that for Shillings, as before, &c.

Example

$$\begin{array}{r|l}
 \frac{1}{3} & 7368 \text{ lb at } 2 \text{ qrs.} \\
 \hline
 \frac{1}{8} & 2486 \\
 \hline
 \frac{1}{10} & 3017 \\
 \hline
 & \text{l. s.} \\
 & 15 \quad 7
 \end{array}$$

$$\begin{array}{r|l}
 \frac{1}{3} & 8247 \text{ lb at } 2 \text{ qrs.} \\
 \hline
 \frac{1}{8} & 2782 \quad 2 \text{ qrs.} \\
 \hline
 \frac{1}{10} & 3417 \quad 9 \text{ d.} \\
 \hline
 & \text{l. s. d.} \\
 & 17 \quad 7 \quad 9 \text{ facit}
 \end{array}$$

7. When the Price of the Integer is 3 Farthings, then take half the given Number for Three-half-pence, and if any Thing remain it is 3 Farthings; then take the 8th for Shillings, as before, &c.

$$\begin{array}{r|l}
 \frac{1}{2} & 4736 \text{ lb at } 3 \text{ qrs.} \\
 \hline
 \frac{1}{8} & 2368 \\
 \hline
 \frac{1}{10} & 916 \\
 \hline
 & \text{l. s.} \\
 & 14 \quad 16 \text{ facit}
 \end{array}$$

$$\begin{array}{r|l}
 \frac{1}{2} & 425 \text{ lb at } 3 \text{ qrs.} \\
 \hline
 \frac{1}{8} & 212 \quad 2 \text{ qrs.} \\
 \hline
 \frac{1}{10} & 265 \quad 3 \text{ qrs.} \\
 \hline
 & \text{l. s. d. qrs.} \\
 & 16 \quad 19 \quad 0 \quad 3 \text{ facit}
 \end{array}$$

Case 2:

8. When the given Price of the Integer is a Part or Parts of a Shilling, (viz. Pence) divide the given Number of Integers (whose Value is sought) by the Denominator of the Fraction, representing the even Part, and the Quote is Shillings (always minding the 7th Rule of the 9th Chapter) and the whole Shillings may be reduced into Pounds by the 3d Rule of



of this Chapter. *Example.* Let it be required to find the Value of 438<sup>lb</sup> at 3<sup>d</sup>. per <sup>lb</sup> I consider 3<sup>d</sup>. is  $\frac{3}{4}$  of a Shilling, and 438<sup>lb</sup> will cost so many 3 Pences, wherefore I divide 438 by 4, the Denominator of  $\frac{3}{4}$ , and the Quotient is 109 Shillings, and 2 remains, which is two 3<sup>d</sup>. or 6<sup>d</sup>. the whole Value is 5<sup>l</sup>. 9<sup>s</sup>. 6<sup>d</sup>. as by the following Work appeareth.

$$\begin{array}{r|l} \frac{1}{4} & 438^{\text{lb. at } 3^{\text{d.}}} \\ \hline \frac{1}{20} & 109 \text{ --- } 6^{\text{d.}} \end{array}$$

*Facit* 5 9

More Examples follow.

$$\begin{array}{r|l} \frac{1}{2} & \text{lb. d.} \\ \hline \frac{1}{20} & 3574 \text{ at } 6 \text{ per } 1. \\ & 178 | 7 \\ \hline & 89^{\text{l. } 7^{\text{s. facit}}} \end{array}$$

$$\begin{array}{r|l} \frac{1}{3} & \text{lb. d.} \\ \hline \frac{1}{20} & 438 \text{ at } 4 \text{ per } 1. \\ & 146 \\ \hline & 7^{\text{l. } 6^{\text{s. facit}}} \end{array}$$

$$\begin{array}{r|l} \frac{3}{4} & \text{lb. d.} \\ \hline \frac{1}{20} & 879 \text{ at } 3 \text{ per } 1. \\ & 219 \quad 9^{\text{l.}} \\ \hline & 10^{\text{l. } 19^{\text{s. } 9^{\text{d.}}}} \end{array}$$

$$\begin{array}{r|l} \frac{1}{8} & \text{lb. d.} \\ \hline \frac{1}{20} & 5316 \text{ at } 2 \text{ per } 1. \\ & 88 | 6 \\ \hline & 44^{\text{l. } 6^{\text{s. facit}}} \end{array}$$

$$\begin{array}{r|l} \frac{1}{8} & \text{lb. d.} \\ \hline \frac{1}{20} & 6389 \text{ at } 1\frac{1}{2} \text{ per } 1. \\ & 79 | 8 \quad 7^{\text{d. } \frac{1}{2}} \\ \hline & 39^{\text{l. } 18^{\text{s. } 7\frac{1}{2}^{\text{d.}}}} \end{array}$$

$$\begin{array}{r|l} \frac{1}{12} & \text{lb. d.} \\ \hline \frac{1}{20} & 818 \text{ at } 1 \text{ per } 1. \\ & 67 | 8 \quad 2 \\ \hline & 3^{\text{l. } 8^{\text{s. } 2^{\text{d. facit}}}} \end{array}$$

If the Learner is minded to try the Fruitfulness of his Genius, he may frame as many Examples as he thinks fit and work them as before

9. If the Price of the Integer be Pence under 12, and yet not an even Part, then it may be divided into even Parts, and so the Parts of the given Numbers taken accordingly and added together; as if it were 5<sup>d</sup>. which is 3<sup>d</sup>. and 2<sup>d</sup>. viz.  $\frac{3}{4}$  and  $\frac{1}{2}$  of a Shilling, first take  $\frac{3}{4}$  of the given Number, and then  $\frac{1}{2}$  thereof, and add them together and their sum is the Answer in Shillings, still observing Rule 7. of Chap. 9. for the Remainder, (if any be) then bring the Shillings into pounds, by the 3<sup>d</sup> Rule foregoing. Likewise 7<sup>d</sup>. is  $\frac{7}{8}$  and  $\frac{1}{4}$ , so 9<sup>d</sup>. is  $\frac{9}{8}$  and  $\frac{1}{4}$ , and 10<sup>d</sup>. is  $\frac{5}{4}$  and  $\frac{1}{2}$ , and 11<sup>d</sup>. is  $\frac{11}{8}$  and  $\frac{1}{4}$  and  $\frac{1}{8}$  of a Shilling; or else many Times your Work may be shortened thus, viz. when the said given Price is to be divided into even Parts of a Shilling, or of a Pound, after you have taken the first even Part, the other may be an even Part of that Part, as in the next Example, where is given 439<sup>lb</sup> at 5<sup>d</sup>. per <sup>lb</sup> now I may divide

divide it thus, viz. into 4d. and 1d. and 4d. being  $\frac{1}{3}$  of a Shilling, and 1d. being  $\frac{1}{4}$  of 4d. I first take  $\frac{1}{3}$  of 439l. and it gives 146s. 4d. and for the 1d I take  $\frac{1}{4}$  of 146s. 4d. which is 36s. 7d. which in all come to 9l. 2s. 11d. Examples follow.

	l.	d.
	439	at 6 per l.
$\frac{1}{3}$	146	4
$\frac{1}{4}$	36	7
	18	2 11
	9l. 2s. 11d. facit	

	Ells	d.
	587	at 7 per Ell
$\frac{1}{3}$	195	8
$\frac{1}{4}$	146	9
	34	2 5
	17l. 2s. 5d. facit	

	yds.	d.
	836	at 8 per yd.
$\frac{1}{3}$	278	8
$\frac{1}{4}$	278	8
	55	7 4
	27l. 17s. 4d. facit	

	yds.	d.
	417	at 9 per yd.
$\frac{1}{3}$	208	6
$\frac{1}{4}$	104	3
$\frac{1}{20}$	31	2 9
	15l. 12s. 9d. facit	

	Ells	d.
	386	at 10
$\frac{1}{3}$	193	
$\frac{1}{4}$	128	8
	32	1 8
	16l. 1s. 8d. facit	

	l.	d.
	534	at 11
$\frac{1}{3}$	178	
$\frac{1}{4}$	178	
$\frac{1}{20}$	133	6
	48	9 6
	24l. 9s. 6d. facit	

10 When the Price of the Integer is Pence and Farthings, if it make an even Part of a Shilling, work as before; but if they are uneven, as Penny Farthing, Penny three Farthings, 2d. 1qr. or 2d. 3qrs. 3d. 3qrs. or the like, then first work for some even Part, and then consider what Part the rest is of that even Part and divide that Quotient thereby, then add them together, and reduce them to Pounds as before. Example, 3470lb

	l.	d.	qr.
	3470	at 1	1
$\frac{1}{12}$	289	2	
$\frac{1}{4}$	72	3	2
$\frac{1}{20}$	36	1 5	2
	l. s. d. qrs.		
	18	1	5 2

at 1d. 1qr. per lb; first I work for the Penny by dividing 3470lb by 12, for 1d. is  $\frac{1}{12}$  of a Shilling, and the Quote is 289s. 2d. then I conceive that one Farthing is the  $\frac{1}{4}$  of a Penny, and the Value at one Farthing will be  $\frac{1}{4}$  of the Value at a Penny, and therefore I take  $\frac{1}{4}$  of 289s. 2d. which is 72s. 3d. 2qrs and add them together, and they are 18l. 1s. 5d. 2qrs. as by the Margent.

	lb.	d.
$7\frac{1}{2}$	4360 at $1\frac{1}{4}$	
$\frac{1}{4}$	363	4
	90	10
	454	2
	22l. 14s. 2d. facit	
$1\frac{1}{2}$	485lb. at $2\frac{1}{4}$ d.	
$\frac{1}{2}$	80	10
	10	$1\frac{1}{4}$
	90	$11\frac{1}{4}$
	4l. 10s. $11\frac{1}{4}$ d. facit	
$1\frac{1}{2}$	654lb. at $2\frac{1}{4}$ d.	
$\frac{1}{2}$	109	
	27	3d.
	136	
	6l. 16s. 3d. facit	

	yds.	d.
$1\frac{1}{2}$	573 at $1\frac{1}{4}$	
$\frac{1}{2}$	71	$7\frac{1}{4}$
	11	$11\frac{1}{4}$
	813	$6\frac{1}{4}$
	4l. 3s. $6\frac{1}{4}$ d. facit	
$1\frac{1}{2}$	520 yds. at $7\frac{1}{2}$ d.	
$\frac{1}{2}$	260	
	65	
$2\frac{1}{2}$	325	
	16l. 5s. facit	
$1\frac{1}{2}$	137 yds. at $10\frac{1}{2}$ d.	
$\frac{1}{2}$	68	6d.
$\frac{1}{2}$	34	3
$\frac{1}{2}$	17	$1\frac{1}{2}$
$1\frac{1}{2}$	1119	$10\frac{1}{2}$ d.
	5l. 19s. $10\frac{1}{2}$ d. facit	

## Case 4

11. When the Price of the Integer is 2s. then cut off the Figure in the Place of Units of the given Number, and double it for Shillings, and the Figures on the other hand are Pounds. *Example.* 436 Yards at 2s. per Yard; cut off the last Figure 6, and double it, it makes 12s. and the two other Figures, viz. 43l. 12s. 43. are so many Pounds; so that their Value is 43l. 12s. as per Margent.

12. Hence it is evident, that when the given Price of an Integer is an even Number of Shillings, then if you take half of that (even Number of Shillings) and multiply the given Number of Integers thereby, doubling the First Figure of the Product and setting it apart for Shillings, the rest of the Product will be Pounds, which Pounds and Shillings are the Value sought. *Example.* What cost 536 Yards at 8s. per Yard? To resolve which, I take half of 8s. (the Price of a Yard) which is 4, and multiply 536 thereby, saying, 4 times 6 is 24, then I double the first Figure 4 makes 8 for Shillings, and carry 2 to the next Product, &c. I find the rest of the Product to be 214, which I note for Pounds; so that the Value of 536 Yards at 8s. per Yard, is 214l. 8s. as per the Margent. Other Examples of the same Kind may be wrought after the same Manner.

536 yds. at 8s.  
4  
214l. 8s.

56 yds. at 6s. per yard
16l. 16s. facit
123 yds. at 4s. per yard
24l. 12s. facit
48 Ells at 8s. per Ell
19l. 4s. facit
84 yds. at 10s. per yard
42l. facit

420 yds. at 12s. per yd.
252l. facit
326 yds. at 14s. per yd.
228l. 4s. facit
48 yds. at 16s. per yard
38l. 8s. facit
52 yds. at 18s. per yard
46l. 16s. facit

13. If the given Price of the Integer is an odd Number of Shillings, then work first for the even Number of Shillings, by the last Rule, and for the odd Shilling take  $\frac{1}{20}$  of the given Number of Integers, according to the 3d Rule of this Chapter, and add them together, and you have your Desire. Examples follow.

Yds.	s.
422 at 3 per Yard	
1.	s.
42	4
21	2
63	0 facit
Ells	s.
516 at 7 per Ell	
1.	s.
154	16
25	16
180	12 facit

Ells	s.
431 at 13	
1.	s.
258	12
21	11
280	03 facit
Ells	s.
324 at 17 per Ell	
1.	s.
259	04
16	04
275	08 facit

14. Except when the given Price of the Integer is 5s. for then it is sooner answered, by taking  $\frac{1}{4}$  of the given Number whose Value is sought, as in the following Example.

$\frac{1}{4}$	Yds.	s.	$\frac{1}{4}$	Ells	s.
	436 at 5 per Yard			206 at 5 per Ell	
	109l. facit			51l. 10s. facit	

Case 5.

15. When the given Price of an Integer is Shillings and Pence, &c. making an even Part of a Pound, then divide the given Number of Integers, whose Value you seek, by the Denominator of that Fraction representing that even Part. As for Example. What is the Price of 384 Yards at 6s. 8d. per Yard? Here I consider that 6s. 8d. is  $\frac{1}{3}$  of a Pound, wherefore divide 384 by 3, and the Quote is the Answer, viz. 128l. to that 384 Yards at 6s. 8d. per Yard, amount to 128l. as per Margin, still observing the 7th Rule of the 9th Chapter.

More Examples follow.

$\frac{1}{3}$	438 Ells at 6s. 8d.	$\frac{1}{8}$	443 Yards at 2s. 6d.
	<hr/> 146l. facit		<hr/> 55l. 7s. 6d. facit
$\frac{1}{8}$	525 at 3s. 4d.	$\frac{1}{2}$	726 Yards at 1s. 8d.
	<hr/> 87l. 10s. facit		<hr/> 60l. 10s. facit

16. When the given Value of the Integer is Shillings and Pence, and not an even Part of a Pound, yet many times it may be divided into Parts, (*viz.* 6s. 6d. is 4s. and 2s. 6d.) For the 4s. work according to the 12th Rule foregoing, and for the 2s. 6d. take the eighth Part of the given Number, and add them together, then their Sum is the Value required.

So 8s. 6d. will be divided into 6s. and 2s. 6d. and the Price of the given Number may be found out as before, &c. Examples follow.

$\frac{1}{3}$	yds. s. d.	$\frac{1}{8}$	Ells s. d.
	386 at 8 8	s.	427 at 8 6
	<hr/> 128l. 13 4	6	<hr/> 128l. 2 0
$\frac{1}{10}$	38 12 0	$\frac{1}{8}$	53 7 6
	<hr/> 167l. 5s. 4d. facit		<hr/> 181l. 9s. 6d. facit
	Ells s. d.		yds. s. d.
s.	440 at 5 4	s.	386 at 14 8
	<hr/> 54 0	8	<hr/> 154l. 8 0
$\frac{1}{8}$	90 0	$\frac{1}{3}$	128 13 4
	<hr/> 144l. facit		<hr/> 283l. 1s. 4d. facit

17. When the given Price of an Integer is Shillings and Pence, and you cannot readily divide them according to the last Rule, then multiply the given Number whose Value you seek, by the Number of Shillings in the Price of the Integer, and then for the Pence work by the 8th Rule foregoing; then add the Numbers together, and their Sum is their Value sought in Shillings; as for Example. What is the Value of 392 Yards at 6s. 9d. per Yard. Here 6s. 9d. cannot be made an even Part, nor indeed can it be divided into even Parts of a Pound; wherefore I multiply the given Number of Yards 392 by 6 for the 6s. the Product is 2352s. then for the 9d. I divide it into 6d. and 3d. and work for them by the 8th Rule foregoing, and at last add the Shillings together, they make 2646s. and by the third they are reduced to 132l. 6s. the Value of 392 Yards at 6s. 9d. per Yard. See the Work.



- 392 yds. at 6s. 9d.

2352

196

98

2646

1324 6s. facit

Other Examples follow.

	lb.	s.	d.
	480 at 4	10	
d	s.		
6	4	1920	
4	$\frac{1}{2}$	240	
	$\frac{1}{4}$	160	
		2320	
		116l. facit	

	Ells	s.	d.
	732 at 12	7	
	12		
	8784		
	4 $\frac{1}{2}$	244	
	34	7183	
		9211	
		460l. 11s. facit	

In like manner may Variety of other Examples be wrought.

18. When the given Price of the Integer is Shillings, Pence and Farthings, then multiply the given Number of Integers, by the Numbers of Shillings contained in the Value of the Integer, and for the Pence and Farthings follow the 10th Rule of this Chapter.

Example.

	Ells	s.	d.
	438 at 8	6 $\frac{3}{4}$	
s.			
8		3504	
$\frac{1}{2}$		219	
$\frac{1}{8}$		27	4 $\frac{1}{2}$
		37510	4 $\frac{1}{2}$
		Fac. 187l. 10s. 4 $\frac{1}{2}$ d.	

	Ells	s.	d.
	136 at 9	2 $\frac{1}{2}$	
s.			
9		1224	0
$\frac{1}{2}$		22	8
$\frac{1}{8}$		5	8
		1252	14
		Fac. 62l. 12s. 4d.	

	Ells	s.	d.
	370 at 14	2 $\frac{1}{2}$	
	1480		
s.			
14		5180	
$\frac{1}{2}$		61	8
$\frac{1}{8}$		15	10
$\frac{1}{4}$		7	8
		5204	9 $\frac{1}{2}$
		Fac. 263l. 4s. 9 $\frac{1}{2}$ d.	
	Ells	s.	d.
	431 at 2	4 $\frac{1}{2}$	
s.			
2		862	
$\frac{1}{2}$		107	0
$\frac{1}{8}$		53	0
		1023	7 $\frac{1}{2}$
		Fac. 51l. 3s. 7 $\frac{1}{2}$ d.	

## Case 6.

19. When the give Value of the Integer is Pounds, then multiply the Number of Integers, whole Value is sought, by the Price of the Integer, and the Product is the Answer in Pounds.

## Examples.

C. l.  
42 at 2 per C.

84l. facit

C. l.  
30 at 3 per C.

90l. facit

C. l.  
13 at 8 per C.

104l. facit

C. l.  
48 at 12 per C.

576l. facit

## Case 7.

20. If the Price of the Integer is Pounds and Shillings, then for the Pounds work as in the last Rule, and for the Shillings as in the 12th and 13th Rules foregoing; then add the Numbers produced from them both, and the Sum the Value sought.

## Examples.

	C.	l.	s.
	46	at 2	4
2l.	92	s.	
4s.	9	4	
	101	4	
	Gross	l.	s.
	58	at 3	7
3l.	174	s.	
6s.	17	8	
1s.	2	18	
	194l.	0s.	facit

	Gross	l.	s.
	82	at 4	10
4l.	328		
10s.	41		
	369l.	facit	
	Gross	l.	s.
	26	at 3	15
3l.	78		
14s.	18	4	
1s.	1	6	
	97l.	10s.	facit

21. When the given Price of an Integer consists of Pounds Shillings, Pence and Farthings, then work for the Shillings, Pence, and Farthings, according to the 18th Rule of this Chapter, and find the total Value of the given Number as if there were no Pounds, then work with the Pounds, according to the 19th Rule of this Chapter, and add the Numbers thus found, and their Sum is the total Value required.

## Examples

Examples of this Rule follow.

	C.	l.	s.	d.
	213 at 1	13	4	$\frac{1}{2}$
	639			
	213			
13s.	2769		d.	
3d.	53		3	
$\frac{1}{2}$ d.	26		$7\frac{1}{2}$	
	28418		$10\frac{1}{2}$	
1l.	142l. 08s. $10\frac{1}{2}$ d.			
	213			
	355l. 08s. $10\frac{1}{2}$ d. facit			
	Gross l. s. d.			
	416 at 2	9	$3\frac{3}{4}$	
9s.	3744			
3d.	104			
$\frac{1}{4}$ d.	26			
	38714			
	193l. 14s.			
2l.	832			
	1025l. 14s. facit			

C	l.	s.	d.
37 at 3	8	$10\frac{1}{2}$	
276 d.	8s.		
18 6	6d.		
9 3	3d.		
4 $7\frac{1}{2}$	$1\frac{1}{2}$ d.		
32 8 $4\frac{1}{2}$			
16l. 8s. $4\frac{1}{2}$ d.			
111	3l.		
127l. 8s. $4\frac{1}{2}$ d. facit			
Gross l. s. d.			
48 at 3	15	$11\frac{1}{2}$	
240			
48			
720	15s.		
24	6d.		
16	4d.		
6	$1\frac{1}{2}$ d.		
7616			
38 6			
144	3l.		
182l. 6s. facit			

22. When there is given the Value of an Integer, and it is required to know the Value of many such Integers together, with  $\frac{1}{4}$  or  $\frac{1}{2}$  or  $\frac{3}{4}$  of an Integer, then first (by the former Rules) find out the Value of the given Number of Integers, and then for  $\frac{1}{4}$  of an Integer, take  $\frac{1}{4}$  of the given Value of the Integer, or for  $\frac{1}{2}$  take  $\frac{1}{2}$  of the given Value of the Integer; and for  $\frac{3}{4}$  first take the half of the given Value, and then half of that half, setting each Part under the Precedent, then adding them together, their Sum will be the required Value of the Integers and their Parts.

Example.

What is the Value of 116 $\frac{1}{2}$  Yards, at 4s. 6d per Yard? To give an Answer; first, I work for the Value of 116 Yards, by the 15th Rule foregoing, and then for the half Yard, I take half of 4s. 6d. which is 2s. 3d. and add to the rest found as before, then is that Sum the total Value of 116 $\frac{1}{2}$  Yards at 4s. 6d. per Yard, which I find to amount to 26l. 4s. 3d. as by the Work in the Margent. And all other Examples of this Kind are wrought the same Way.

Other

Other Examples follow.

324 $\frac{1}{2}$  yds. at 4s. 10d.

1296	4s.
162	6d. $\frac{1}{2}$
108	4d. $\frac{1}{3}$
1	2 $\frac{1}{2}$ d. $\frac{1}{4}$ yd.

156 7s. 2 $\frac{1}{2}$ d.78l. 7s. 2 $\frac{1}{2}$ d. facit228 $\frac{1}{2}$  Ells at 12s. 11d.

2756	12s.
76	4d. $\frac{1}{3}$
76	4d. $\frac{1}{3}$
57	3d. $\frac{1}{3}$
6	5 $\frac{1}{2}$ d. $\frac{1}{4}$ Ell
3	2 $\frac{3}{4}$ d. $\frac{1}{4}$ Ell

295l 4 8 $\frac{1}{2}$ d.147l. 14s. 8 $\frac{1}{2}$ d. facit720 $\frac{1}{2}$  yds. at 6s 8d.

240l. 3s. 4d. facit

C. qrs. lb l. s.

28 3 14 at 1 10

28l. 1l.

14 10 $\frac{1}{2}$ 15s. 1 $\frac{1}{2}$ C.7s. 6d. 1 $\frac{1}{2}$ C.

3s 9d. 14l.

43l. 6s. 3d. facit

Many more Questions may be stated, and several other Rules of Practice may be shewn, according to the Methods of diverse Authors, but what have been delivered here are sufficient for the practical Arithmetician in all Cases whatsoever.

## C H A P. XXVII

## Barter.

1. **BARTER** is a Rule among Merchants, which (in the Exchange of one Commodity for another) informs them so to proportion their Rates as that neither may sustain Loss.

2. To resolve Questions in Barter, will not be Difficult to him that is acquainted with the *Golden Rule* or *Rule of Three*, it being altogether used in resolving such Questions.

*Quest. 1.* Two Merchants (*viz.* A and B) barter, A hath 13C. 3qrs. 14lb of Pepper, at 2l. 16s. per C. and B hath Cotton at 9d. per lb I demand how much Cotton B must give A for his Pepper?

*Ans. 9C. 1qr.*

First find by the *Rule of Three*, or the Rules of Practice foregoing, how much the Pepper is worth, saying, if 1C. cost 2l. 16s. what will 13C. 3qrs. 14lb cost?

*Answer 38l. 17s.*

Secondly, by the *Rule of Three*, say if 9d. buy 1lb of Cotton, how much will 38l. 17s. buy?

*Answer. 9 $\frac{1}{2}$ C. and so much Cotton must B give to A for*

13C. 3grs. 1½ lb of Pepper, at 2l. 16s. per C. when the Cotton is worth 9d. per lb.

Quest. 2. *A* and *B* barter, *A* hath 120 Yards of Broadcloth worth 6s. per Yard, but in the Barter he will have 8s. per Yard; *B* hath Shalloon worth 4s. per Yard. Now demand how many Yards of Shalloon *B* must give *A* for his Broadcloth, making his Gain in Barter equal to that of *A*?

Answer 180 Yards of Shalloon.

First (as in the last Question) find out how *B* ought to sell his Shalloon in Barter, viz. say if 6s. require 8s. what will 4s. require?

Answer 5s. 4d.

Thus you see that *B* must sell his Shalloon in Barter at 5s. 4d. if *A* sell his Broadcloth at 8s. per Yard.

It remaineth now to find out how much Shalloon *B* must give for 120 Yards of Broadcloth; which resolved after the Method in the first Question of this Chapter, is found to be 180, and so many Yards of Shalloon must *B* give *A* for the 120 Yards of Broadcloth.

Quest. 3. *A* and *B* bartered, *A* had 14C. of Sugar, worth 6d. per lb, for which *B* gave him 1C. 3grs. of Cinnamon; demand how *B* rated his Cinnamon per lb?

Answer 4s. per lb.

Quest. 4. *A* and *B* barter, *A* hath 4 Tun of Brandy, worth 37l. 16s. ready Money, but in Barter he hath 50l. 5s. per Tun, and *B* giveth 21C. 2grs. 11½ lb of Ginger for the 4 Tun of Brandy; I desire to know how much *B* sold his Ginger for in Barter per C. and how much it is worth in ready Money?

Answer for 9l. 6s. 8d. in Barter, and it is worth 7l. per C. in ready Money.

Quest. 5. *A* and *B* barter, *A* hath 320 Dozen of Candles at 4s. 6d. per Dozen, for which *B* giveth him 30l. in Money, and the rest in Cotton at 8d. per lb; I demand how much Cotton he must give him more than the 30l.

Answer 11C. 1gr.

## C H A P. XXVIII.

### Questions in Loss and Gain.

1 *A* Merchant bought 436 Yards of Broadcloth for 8s. 6d. per Yard, and selleth it again at 10s. 4d. per Yard; now I desire to know how much he gained in the sale of the 436 Yards? Answer 39l. 19s. 4d.

First,



154 *Questions in Loss and Gain.* Chap. 28

First, find out by the *Rule of Three*, or by *Practice*, how much the Cloth cost him at 8s. 6d. per Yard, which I find to be 185l. 6s. then by the same Rule find out how much he sold it for, viz. 225l. 5s. 4d. then subtract 185l. 6s. which it cost him, from 225l. 5s. 4d. which he sold it for, and there remaineth 39l. 19s. 4d. for his Gain in the Sale thereof.

Otherwise, it may sooner be resolved thus; first find out how much he gained per Yard, viz. subtract 8s. 6d. which he gave per Yard, from 10s. 4d. which he sold it for per Yard, the Remainder is 1s. 10d. for his Gain per Yard. Then say,

If 1 Yard gain 1s. 10d. what will 436 Yards gain? The Answer by *Practice* or the *Rule of Three*, is 39l. 19s. 4d. as was found before.

*Quest. 2.* A Draper bought 124 Yards of Holland Cloth for which he gave 31l. I desire to know how he must sell it per Yard to gain 10l. 6s. 8d. in the whole Sale of 124 Yards?

*Answer* at 6s. 8d. per Yard.

Add the Price which it cost him (viz. 31l.) to his intended Gain, (viz. 10l. 6s. 8d.) the Sum is 41l. 6s. 8d. Then say,

If 124 Yards require 41l. 6s. 8d. what will 1 Yard require? By the *Rule of three* I find the Answer to be 6s. 8d.

*Quest. 3.* A Grocer bought 3C. 1<sup>qr</sup>. 14lb of Cloves which cost him 2s. 4d. per lb, and sold them for 52l. 14s. I desire to know how much he gained in the whole?

*Answer* 8l. 12s.

*Quest. 4.* A Draper bought 86 Kerseys for 129l. I demand how he must sell them per Piece to gain 15l. in laying out 100l. at that Rate?

*Answer* 1l. 14s. 6d. per Piece; for,

As 100l. is to 115l. so is 129l. to 148l. 7s.

So that, by the Proportion above, I have found how much he must receive for the 86 Kerseys, to gain after the Rate of 15 per Cent. Then to find how he must sell them per Piece, I say,

As 86 Pieces are to 148l. 7s. so is 1 Piece to 1l. 14s. 6d. which is the Number sought.

*Quest. 5.* A Grocer bought 4<sup>1</sup>/<sub>2</sub>C. of Pepper for 15l. 17s. 4d. and (it proving to be dammified) is willing to lose 10s. per Cent. I demand how he must sell it per lb?

*Answer* 7d. per lb.

Subtract 12l. 10s. the Loss of 100l. from 100l. and there remains 87l. 10s. Then say,

28 Chap. 28. *Questions in Loss and Gain.* 155

As 100*l.* is to 87*l.* 10*s.* so is 15*l.* 17*s.* 4*d.* to 13*l.* 17*s.* 8*d.* and so much he must sell it all for, to lose after the Rate propounded. Then to know how he must sell it *per lb* I say,

As 4½*C.* is to 13*l.* 17*s.* 6*d.* so is 1*lb* to 7*d.*  
*Quest. 6.* A Plummer sold 10 Fodder of Lead (the Fodder containing 19½*C.*) for 204*l.* 15*s.* and gained after the Rate of 12*l.* 10*s.* *per* 100*l.* I demand how much it cost him *per C*?  
*Answer* 18*s.* 8*d.*

To resolve this Question, add 12*l.* 10*s.* (the Gain *per Cent*) to 100*l.* and it makes 112*l.* 10*s.* Then say,  
 As 112*l.* 10*s.* is to 100*l.* so is 204*l.* 15*s.* to 182*l.* which 182*l.* is the Sum it cost him in all; then reduce your 10 Fodders to Half Hundreds, and it makes 390. Then say,

As 390 Half Hundreds is to 182*l.* so is 2 Half Hundreds to 18*s.* 8*d.* the Price of 2 Half Hundreds, or 1 *Cwt.* and so much it stood him in *per Cwt.*

*Quest. 7.* A Merchant bought eight Tuns of Wine, which, being sophisticated, he selleth for 400*l.* and loseth after the Rate of 12*l.* in receiving 100*l.* Now I demand how much it cost him *per Tun*, and how he selleth it *per Gallon* to lose after the said Rate?

*Ans.* It cost him 56*l.* *per Tun*, and he must sell it at 3*s.* 11*d.* 2½*grs* Gallon, to lose 12*l.* in receiving 100*l.*

To resolve this Question, I consider, in the first Place, that in receiving 100*l.* he loseth 12*l.* therefore 100*l.* comes in for 112*l.* laid out; wherefore, to find out how much he laid out for the whole, I say,

As 100*l.* is to 112*l.* so is 400 to 448*l.* and so much the 8 Tun cost him. Then to find out how much it cost *per Tun*, I say,

As 8 is to 448*l.* so is 1 to 56*l.* the Price it cost *per Tun.*

Now to find how he must sell it *per Gallon*, reduce the 8 Tuns into Gallons, they make 2016 Then say,

As 2016 Gallons is to 400*l.* so is 1 Gallon to 3*s.* 11*d.* 2½*grs.* the Price he must sell it at *per Gallon* to lose as foresaid.

*Quest. 8.* A Merchant bought 8 Tun of Wine, which being sophisticated he is willing, to sell for 400*l.* and loseth at that Rate 12*l.* in laying out 100*l.* upon the same; now I demand how much it cost him *per Tun*?

Here I consider, that for 100*l.* laid out he received but 88*l.* wherefore to find what 8 Tuns cost him, I say,

As 88*l.* is to 190*l.* so is 400*l.* to 454½*s.* the Price it all cost him. Then to find out how much *per Tun*, I say,

As 8 is to 454½*s.* so is 1 to 56½*s.* or 56*l.* 16*s.* 4½*grs.* *per Tun.*

C H A P.

## C H A P. XXIX.

*Equation of Payments.*

1. **E**QUATION of Payments, is that Rule among Merchants, whereby we reduce the Times for Payment of several Sums of Money, to an equated Time for Payment of the whole Debt, without Damage to Debtor or Creditor; and

*The Rule is*

2. Multiply the Sum of each particular Payment by its respective Time, then add the several Products together, and their sum divide by the total Debt, and the Quotient thence arising is the equated Time for the Payment of the whole Debt.

*Example.*

*Quest. 1.* A is indebted to B in the Sum of 130*l.* whereof 50*l.* is to be paid at 2 Months, and 50*l.* at 4 Months, and the rest at 6 Months; now they agree to make one Payment of the total Sum: The Question is, what is the equated Time for Payment, without Damage to Debtor or Creditor?

To resolve this Question, I multiply each Payment by its Time, *viz.*

50 <i>l.</i> multiplied by 2 Months produceth	100
50 <i>l.</i> multiplied by 4 Months produceth	200
30 <i>l.</i> multiplied by 6 Months produceth	180

The sum of the Products is 480

Then I divide 480 (the Sum of the Products) by 130 (the total Debt) and the Quotient is  $3\frac{2}{3}$  Months for the Time of paying the whole Debt.

*Quest. 2.* A Merchant hath owing to him 1000*l.* to be paid as followeth, *viz.* 600*l.* at 4 Months, 200*l.* at 6 Months, and the rest (which is 200*l.* at 12 Months,) and he agreeth with the Debtor to make one payment of the whole; I demand the Time of Payment without Damage to Debtor or Creditor?

600 <i>l.</i> multiplied by 4 Months is	2400
200 <i>l.</i> multiplied by 6 Months is	1200
200 <i>l.</i> multiplied by 12 Months is	2400

The Sum of the Products is 6000

and the Sum of the Products (6000) being divided by the whole Debt (1000*l.*) quotes 6 Months for the Time of Payment of the whole Debt.

3. The

3. The Truth of the Rule is thus manifest, if the Interest of that Money which is paid by the equated Time (after it is due) be equal to the Interest of that Money which (by the equated Time) is paid so much sooner than it is due at any Rate *per Cent.* then the Operation is true, otherwise not.

*The Proof of the Rule of Equation of Payments.*

*Example.*

In the last Question 600*l.* should have been paid at 4 Months, but it is not discharged till 6 Months (that is 2 Months after it is all due) wherefore its Interest for 2 Months at 6 *per cent. per annum* is 6*l.* and then 200*l.* was to be paid at 6 Months, which is the equated Time for its Payment, therefore no Interest is reckoned for it, but 200*l.* should have been paid at 12 Months, but it is paid at 6 Months, which is 6 Months sooner than it ought, wherefore the Interest of 200*l.* for 6 Months is 6*l.* (accounting 6*l. per cent. per annum*) which is equal to the Interest of 600*l.* for 2 Months, wherefore the Work is right.

*Quest. 3.* A Merchant hath owing him a certain Sum to be discharged at three equal Payments, *viz.*  $\frac{1}{3}$  at two Months,  $\frac{1}{3}$  at four Months, and  $\frac{1}{3}$  at eight Months, the Question is, what is the equated Time for the Payment of the whole Debt?

In Questions of this Nature (*viz.*) where the Debt is divided into unequal Parts) each of its Parts is to be multiplied by its Time, and the Sum of the Products is the Answer.

$\frac{1}{3}$ multiplied by 2 Months produceth	$\frac{2}{3}$
$\frac{1}{3}$ multiplied by 4 Months produceth	$\frac{4}{3}$
$\frac{1}{3}$ multiplied by 8 Months produceth	$\frac{8}{3}$

The Sum of the Product is  $4\frac{2}{3}$  which is  $4\frac{2}{3}$  Months for the equated Time of Payment.

If instead of the Fractions representing the Parts, you had wrought by the Numbers themselves (represented by those Parts) according to the first and second Example, it would have been the same Answer; and suppose the Debt had been 90*l.* then  $\frac{1}{3}$  of it is 30*l.* for each Payment, *viz.* at 2, 4 and 8 Months.

30 <i>l.</i> multiplied by 2 Months produceth	60
30 <i>l.</i> multiplied by 4 Months produceth	120
30 <i>l.</i> multiplied by 8 Months produceth	240

The Sum of the Products is 420

Half Sheet 1

which

which divided by 90 (the whole Debt) quoteth  $4\frac{2}{3}$ , or  $4\frac{2}{3}$  Months, as before.

*Quest. 4.* A Merchant oweth a Sum of Money. to be paid  $\frac{1}{2}$  at 5 Months, and  $\frac{1}{4}$  at 8 Months, and  $\frac{1}{4}$  at 10 Months, and he agreeth with his Creditor to make one total Payment; I demand the Time without Damage to Debtor or Creditor? Work as in the last Question, and you will find the Answer to be 7 Months.

*Quest. 5.* A is indebted to B 640*l.* whereof he is to pay 40*l.* present Money, 350*l.* at 3 Months, and the rest, *viz.* 250*l.* at 8 Months, and they agree to make an equated Time for the whole Payment; now I demand the Time?

In Questions of this Nature (*viz.* where there is ready Money paid) you are, in multiplying, to neglect the Money that is to be paid present, and work with the rest, as is before directed, and divide the Sum of the Products by the whole Debt, and the Quote is the Answer; for here 40*l.* is to be paid present, and hath no Time allowed; and according to the Rule it should be multiplied by its Time, which is 0, therefore 40 times 0 is 0, which neither augmenteth nor diminisheth the Dividend; wherefore to proceed (according to direction) I say.

350 by 3 Months produceth 1050  
250 by 8 Months produceth 2000

The Sum of the Product is 3050

which divided by 640, the whole Debt, the Quote is  $4\frac{2}{3}$  Months, the Time of Payment.

*Quest. 6.* A is indebted to B in a certain Sum, half whereof is to be paid present Money,  $\frac{1}{3}$  at 6 Months, and the rest at 8 Months; now I demand the equated Time for Payment of it all?

*Answer*  $3\frac{1}{3}$  Months is the Time of payment.

*Quest. 7.* A is indebted to B 120*l.* whereof  $\frac{1}{4}$  is to be paid at 3 Months,  $\frac{1}{4}$  at 6 Months, and the rest at 9 Months; what is the equated Time for payment of the whole Sum?

*Ans.* at  $6\frac{1}{4}$  Months.

*Quest. 8.* A is indebted to B 420*l.* which is due at the End of 6 Months, but A is willing to pay him 140*l.* present, provided he can have the Remainder forborn so much the longer, to make Satisfaction for his Kindnets, which is agreed upon; I desire to know what Time ought to be allotted for the Payment of the 280*l.* remaining?

The



The Operation of this Question is left to the Learner, to try his Genius, and who, in this Case, must have an Eye to the Rule of Three.

## C H A P. XXX.

## Exchange.

1. **T**HE Rule of *Exchange* informeth the Merchants how to exchange Monies, Weights or Measures of our Country into (or for) the Monies, Weights or Measures of another Country, and when the Rate, Reason or Proportion betwixt the Money, Weights or Measures of different Countries is known, it will not be difficult for the Practitioner that is well acquainted with the Rule of Proportion (or *Rule of Three*) to resolve any Question, wherein it is required to exchange a given Quantity of the one Kind into the same Value of another Kind.

2. In Questions of Exchange there is always a Comparison made between the two Coins, &c. of two Countries (or Kinds) or of more.

3. In Questions where there is a Comparison made between two Things (whether they be Monies, Weights, &c.) of different Kinds, there may be a Solution found by a single Rule of Three, as by the following Example.

*Quest. 1.* A Merchant at *London* delivered 370*l. ster.* to receive the same at *Paris* in French Crowns, the Exchange  $3\frac{1}{3}$  French Crowns per 1*l. sterling*; I demand how many French Crowns he ought to receive?

In placing the Numbers, observe the 6th Rule of the 10th Chapter, which being done, the given Number will stand thus:

1.	Crowns	1.
1	$3\frac{1}{3}$	370

and being reduced according to the Rules of the 24th

As  $\frac{1}{3}$  is to  $\frac{10}{3}$ , so is  $370$  to  $1233\frac{1}{3}$ .

So that I conclude he ought to receive  $1233\frac{1}{3}$  French Crowns at *Paris* for his 370*l.* delivered at *London*.

*Quest. 2.* A Merchant delivered at *Amsterdam* 587*l. Flemish*, to receive the Value thereof at *Naples* in Ducats, the Exchange  $4\frac{2}{3}$  Ducats per 1*l. Flemish*; I demand how many Ducats he ought to receive?

The Proportion is as followeth:

1.	Ducats	1.	Ducats
As $\frac{1}{3}$ is to $2\frac{2}{3}$ ,	so is	587	to 2817 $\frac{2}{3}$
	12		

So

So I find he ought to receive  $2817\frac{3}{4}$  Ducats at Naples, for the 587l. *Flemish* delivered at Amsterdam.

*Quest. 3.* A Merchant at Florence delivereth 3478 Ducatoons, to receive the Value at London in Pence, the Exchange at  $53\frac{1}{2}d.$  *sterling* per Ducatoon; I demand how much *sterling* he ought to receive?

The Proportion for Resolution is,

Ducats      d.      Ducats      d.

As  $\frac{1}{4}$  is to  $10\frac{1}{2}$ , so is  $3478\frac{3}{4}$  to 186073

which is equal to 775l.  $6\frac{1}{2}$  for the Answer.

4. When there is a Comparison made between more than two different Coins, Weights or Measures, there ariseth ordinarily two different Cases from such a Comparison.

1. When it is required to know how many Pieces of the first Coin, Weight or Measure are equal in Value to a known Number of Pieces of the last Coin, Weight or Measure.

2. When it is required to find out how many Pieces of the last Coin, Weight or Measure are equal in Value to a given Number of the first sort of Coin, Weight or Measure.

An Example of the first Case may be this, *viz.*

*Quest. 4.* If 150 Pence at London are equal to 3 Ducats at Naples, and  $4\frac{2}{3}$  Ducats at Naples make  $34\frac{1}{2}$  Shillings at Brussels; then how many Pence at London are equal to 138s. at Brussels? *facit* 960d.

The Question may be resolved by two single Rules of Three. For first I say,

If  $\frac{1}{4}$  Ducats at Naples make 150d. at London, how many Pence will  $4\frac{2}{3}$  Ducats make? *Answer* 240d.

By the foregoing Proportion we have discovered, that  $4\frac{2}{3}$  Ducats at Naples make 240 Pence at London; and by the Tenor of the Question we see that  $4\frac{2}{3}$  Ducats at Venice make  $34\frac{1}{2}$  Shillings at Brussels; therefore 240d. at London are equal to  $34\frac{1}{2}s.$  at Brussels (for the Things that are equal to one and the same Thing, are also equal to one another) wherefore we have a Way laid open to give a Resolution to this Question by another Single Rule of Three, whose Proportion is,

As  $34\frac{1}{2}s.$  at Brussels is to 240d. at London, so is 138s. at Brussels to 960d. at London; which is the Answer to the second Question.

An Example of the second Case may be this, *viz.*

*Quest. 5* If 40lb. *Averdupois-weight* at London is equal to 36lb weight at Amsterdam, and 90lb. at Amsterdam makes 116lb at Dantzick; then how many Pounds at Dantzick are equal to 112lb *Averdupois-weight* at London?

*Answer*

Answer  $129\frac{2}{5}$  lb at Dantzick.

This Question is likewise answered by two single Rules of Three, viz. First I say,

As 36 lb at Amsterdam is to 40 lb at London.

So is 90 lb at Amsterdam to 100 lb at London

And by the Question you find, that 90 lb at Amsterdam is equal to 116 lb at Dantzick. and therefore 100 lb at London is likewise equal thereunto; wherefore again I say,

As 100 lb at London is to 116 lb at Dantzick.

So is 112 lb at London to  $129\frac{2}{5}$  lb at Dantzick.

By which I find, that  $129\frac{2}{5}$  lb at Dantzick are equal to 112 lb Averdupois-weight at London,

5. There is a more speedy Way to resolve such Questions as are contained under the two Cases before mentioned, laid down by Mr Kersey in the third Chapter of his Appendix to Wingate's Arithmetick, wherein he hath given two Rules for the Resolution of the Questions pertinent to the said Cases.

6. But I shall lay down a general Rule for the Solution of both Cases; and 1st, Let the Learner observe the following Directions in placing of the given Terms, viz.

7. Let there be made 2 Columns, and in these Columns so place the given Terms one over the other as that in the same Column there may not be found 2 Terms of the same Kind one with the other.

Having thus placed the Terms, the general Rule is,

Observe which of the said Columns hath the most Terms placed in it, and multiply all the Terms therein continually, and place the last Product for a Dividend; then multiply the Terms in the other Column continually, and let the last Product be a Divisor; then divide the said Dividend by the said Divisor, and the Quotient thence arising will be the Answer to the Question.

So the Example of the first of the said Cases being again repeated, viz. if 150 Pence at London make 3 Ducats at Naples, and  $4\frac{1}{2}$  Ducats at Naples makes  $34\frac{1}{2}$  shillings at Brussels, then how many Pence at London are equal to 138 shillings at Brussels?

The Terms being placed according to the 7th Rule will stand as followeth.

	A	B	
Pence at London.	150	3	Ducats at Naples.
Ducats at Naples.	$4\frac{1}{2}$	$34\frac{1}{2}$	shillings at Brussels.
shillings at Brussels	138		

Having thus placed the Terms that in neither Column there are not two Terms of one Kind, then observe that

the Columu under A hath most Terms in it, therefore they must be multiplied together for a Dividend, viz. 150 multiplied by  $4\frac{1}{2}$  produceth  $360\frac{1}{2}$ , which multiplied by 138 produceth  $49680\frac{1}{2}$  for a Dividend; then in the Column under B there are 3 and  $34\frac{1}{2}$ , which multiplied together produce  $20\frac{1}{2}$  for a Divisor; then having divided  $49680\frac{1}{2}$  by  $20\frac{1}{2}$ , the Quotient is 960 Pence for the Answer, as before.

Again, Let the Example of the second Case be again repeated, viz. if 40lb *Averdupois-weight* at London make 36lb weight at *Amsterdam*, and 90lb at *Amsterdam* make 116lb at *Dantzick*, then how many Pounds at *Dantzick* are equal to 112lb *Averdupois-weight* at London.

The Terms being disposed according to the 7th Rule foregoing, will stand thus.

	A	B	
lb at London	40	36	lb at Amsterdam.
lb at Amsterdam	90	116	lb at Dantzick.
		112	lb at London.

hereby I find that the Terms under B multiplied together produce 467712 for a Dividend, and the Terms under A, viz. 40 and 90, produce 3600 for a Divisor, and Division being finished, the Quotient giveth  $129\frac{33\frac{1}{2}}{3600}$  lb at *Dantzick* for the Answer.

## C H A P. XXXI.

### Single Position.

1. **N**egative Arithmetic, called the *Rule of False*, is that by which we find out a Truth, by Numbers invented or supposed, either single or double.

2. The Rule of single Position is, when at once, viz. by one false Position, or feigned Number, we find out the true Number sought.

3. In the *Single Rule of False*, when you have made choice of your Position, work it according to the Tenor of the Question, as if it were the true Number sought; and if by the ordering your Position you find either the Result too much or too little, you may then find out the Number sought by this Proportion following, viz.

As the Result of your Position is to the Position, so is the given Number to the Number sought.

Example.

*Quest. 1.* A Person having about him a certain Number Crowns 3id, if a 1th, 3rd and 6th of them were added together

together they would make just 45 Crowns; now I demand the Number of Crowns he had about him?

*Answer* 60 Crowns

To resolve this Question, I suppose he had 24 Crowns (or any other Number that will admit of the like Division) now the 4th of 24 is 6, and the 3rd is 8, and the 6th is 4, all which Parts, (6, 8 and 4) being added together, make but 18, but it should be 45, wherefore I say, by the *Rule of Three*,

As 18 the Sum of the Parts is to the Position 24, so is 45 the given Number, to 60 the true Number sought.

For the 4th of 60 is 15, and the 3rd of 60 is 20, and the 6th of 60 is 10, which added together make 45.

## CHAP. XXXII.

### Double Position.

1. **T**HE Rule of *Double Position* is, when two false Positions are assumed to give a Resolution to the Question propounded.

2. When any Question is stated in *Double* a **X**  
*Position*, make such a Crois as in the Margent.

3. Then make choice of any Number you d **X** c  
think may be convenient for your working, which call your first Position, and place it at the End of the Crois at a; then work with this Position as if it were the true Number sought, according to the Nature of your Question; then having found out your Error, either too much or too little, place it on that side of the Crois at d, then make choice of another Number, of the same Denomination with the first Position (which call your second Position) and place it on the Side of the Crois at b; then work with this Position as with the former, and having found out your Error, either too much or too little, place it on that Side of the Crois at c, and then the Positions will stand at the Top of the Crois, and the Errors at the Bottom, each under his Correspondent Position, and then multiply the Errors into the Position cross wise, that is, multiply the first Position by the second Error, and the second Position by the first Error, and put each Product over its Position.

4. Having proceeded so far, then consider whether the Errors are both alike, that is, whether they are both too much, or both too little; and if they are alike, then subtract the lesser Product from the greater, and set the Remainder for a Dividend; then subtract the lesser Error from



from the greater, and let the Remainder be a Divisor, and the Quotient arising by this Division is the Answer to the Question.

5. But if the Errors are unlike, that is, one too much and the other too little, then add the Products of the Positions and Errors together, and their Sum shall be a Dividend; then add the Errors together, and their Sum shall be a Divisor, and the Quotient arising hence is the Answer.

*Quest. 1.* *A, B and C built a House which cost 76*l.* of which A paid a certain Sum unknown, B paid as much as A and 10*l.* over, and C as much as A and B; now I desire to know each Man's Share in that Charge?*

Having made a Cross, according to the second Rule, I come according to the third Rule to make choice of my first Position, and here I suppose *A* paid 6*l.* which I put upon the Cross as you see, then *B* paid 16*l.* (for it's said he paid 10*l.* more than *A*) and *C* paid 22*l.* (for it's said he paid as much as *A* and *B*) then I add their Parts,

L		A
9		B
19		C
28		
---		
56		Sum
76		
56		
---		
20		Error

and they amount to 44; but it is said they paid 76%. wherefore there is 32 too little, which I note down at the Bottom of the Crois under its Position for the first Error.

2dly, I suppose *A* paid 9l. then *B* paid 19l. and *C* 28.. all which added together make 56, but they should make 76, wherefore the Error of this Position is 20, which I put at the Bottom of the Cross under its Position for the second error; then I multiply the Errors and Position cross-wise, viz. 32 (the Error of the first Position) by 9 (the second Position) and the Product is 288; then I multiply 20 (the Error of the second Position) by 6 the first Position and the Product is 120.

Then (according to the 4th Rule) I subtract the lesser Product from the greater, viz. 120 from 288, because the Errors are both alike, (viz. too little) and there remaineth

32- and the uch Po- ivil- hall An- of uch fire , I ny out aid he L 6 16 22 44 76 44 32 re- ot. 8.. ke ut le- fs- fe- 20 on er ne th 68

168 for a Dividend; then I subtract 22 (the lesser Error) from 32 (the greater Error) and the Remainder is 12 for a Divisor; then I divide 168 by 12, and the Quotient is 14 for the Answer, which is the share of *A* in the Payment.

6, Again, 2dly, if the Errors had been both too big, it had the same Effect, as appeareth by the following Work; for first, I suppose *A* paid 20*l* then *B* paid 30*l*. and *C* 50*l*. which in all is 100*l*. but it should have been no more than 76, wherefore the first Error is 24 too much. Again I suppose *A* paid 18*l*. then *B* must pay 28*l* and *C* must pay 46*l*. which in all is 92*l*. but it should have been but 76*l*.

20 <i>A</i>			<i>A</i> 18
30 <i>B</i>			<i>B</i> 28
50 <i>C</i>			<i>C</i> 46
-----	320 112 432		-----
100 Sum	20	X 18	Sum 92
76 Subtract	8)	(14	Subtract 76
-----	24	X 16	-----
	8		

24 Error Error 16  
wherefore the 2d Error is 16 too much; then I multiply 20 (the first Position) by 16 (the 2d Error) and the Product is 320. Again, I multiply 18 (the 2d position) by 24 (the first Error) and the Product is 432. Then, because the Errors are both too much, I subtract 320 (the lesser Product) from 432 (the greater Product) and there remaineth 112 for a Dividend; likewise I subtract 16 (the lesser Error) from 24 (the greater Error) and the Difference is 8 for a Divisor; then perform Division, and the Quotient is 14, as before, for the Answer.

Again, 3dly, if the Errors had been the one too big and the other too little, Respect being had to the fifth Rule foregoing, the Answer would have been the same, as thus, I take for my first Position 6, and then the Error 32 too little; then I take for my second Position 18, and then the Error is 16 too much; then I multiply the Positions and Errors crosswise, and the Products are 96 and 576, and because the Errors are unlike, viz. one too big and another too little, I add the Products 96 and 576 together, and their Sum is 672 for a Dividend; I likewise add the Errors 32 and 16 together, and their Sum is 48 for a Divisor; then having finished Division, I find the Quotient to be 14, which is the Answer, as was found out at the two several Trials before.

96 672 576			
6	X	18	
48)		(14	
32	X	16	
48			

For Proof of this Work, I say,

If <i>A</i> paid	----	7
Then <i>B</i> paid 14 and 10 (that is)		14
Then <i>C</i> paid 14 and 24 (that is)		24
		38

*111* The Sum of all is 76  
which is the total Value of the Building, and equal to the given Number.

Those who desire to see the Demonstration of this Rule, let them read the 7th Chapter of Mr. Kersey's Appendix to Mr. Wingate's Arithmetic, Pitiscus in the 5th Book of his *Trigonometria*, or Mr. Oughtred in his *Clavis Mathematica*.

*Quest. 2.* Three Persons, *A*, *B* and *C*, thus discoursed together concerning their Age; quoth *A*, I am 18 Years of Age; quoth *B*, I am as old as *A* and half *C*, and quoth *C*, I am as old as you both, if your Years were added together; now I desire to know the Age of each Person?

*Answer* *A* is 18, *B* is 54, and *C* is 72 Years of Age.

*Quest. 3.* A Father lying at the Point of Death, left to his three Sons, viz. *A*, *B* and *C*, all his Estate in Money, and divided it as followeth, viz. to *A* he gave half, wanting 4*l.* to *B* he gave  $\frac{1}{3}$  and 14*l.* over, and to *C* he gave the Remainder, which was 82*l.* less than the share of *B*; now I demand what was the Sum left, and each Man's Part?

*Answer.* The Sum bequeathed was 588*l.* whereof *A* had 250*l.* *B* had 210*l.* and *C* had 128*l.*

*Quest. 4.* Two Persons, viz. *A* and *B*, had each in their Hands a certain Number of Crowns, and *A* said to *B*, if you give me one of your Crowns, I shall have five times as many as you; then said *B* to him again, if you give me one of yours, then we shall each of us have an equal Number; now I demand how many Crowns had each Person?

*Answer.* *A* had 4, and *B* had 2 Crowns.

*Quest. 5.* What Number is that unto which if I add  $\frac{1}{4}$  of itself, and from the Sum subtract  $\frac{1}{8}$  of itself, the Remainder will be 216?

*Answer* 192.

Many more Questions may be added, but these well understood will be sufficient (even for the meanest Capacity) for the Resolution of any other Question pertinent to this Rule.

There may be an Objection made, because we have not treated particularly upon Interest and Rebate; but the Operation

## Additional Questions.

67

Operations of such Questions being more applicable to Decimals, are omitted, till we come to acquaint the Learner therewith.

## L A U S D E O S O L I.

A few Additional Questions for the Exercise of Learners, in the several foregoing Rules.

1. **A**N old whimsical Gentleman having 5 Daughters, left them a considerable Fortune which he bequeathed in manner following. The Sum of the Fortunes of the four eldest was 25000*l*. the Sum of the four last 33000*l*. the Sum of three last with the first 30000*l*. the Sum of the three first with the last 28000*l*. and the Sum of the two last with the two first 32000*l*. Now the Question is how much was each particular Daughter's Fortune?

2. **B** and **C** working jointly can finish a Boat in 18 Days; with the Assistance of **A** they can do it in 11 Days. Now in what Time could **A** perform the same, working alone?

3. **X**, **Z** and **T**, working together, can complete a Staircase in 12 Days, **Z**'s Man enough to do the same alone in 24 Days, and **X** in 34. Now in what Time could **T** alone get it done?

4. What Number is that, to which, if  $\frac{3}{10}$  of  $\frac{18}{19}$  of  $\frac{11}{13}$  be added, the Total will be 1?

5. It is proposed by an elderly Person in Trade, to admit a sober industrious young Fellow to a Share in the Business; and, to encourage him offers, that if his Circumstances will allow him to advance 100*l*. his Pay shall be 40*l*. per annum; if he be able to advance 200*l*. in Stock, he shall have 55*l*. a Year; and if 300*l*. he shall receive 70*l*. a Year. In this proposal I desire to know what is allowed the Young Man for his Attendance simply?

6. If 15 Birds cost in all 5*s*. whereof there were Partridges at 7*d*. Quails at 5*d*. and Larks at 2*d*. How many were there of each?

7. A Mixture was made of 10 Gallons of Wine, 4 Gallons of Brandy and 12 Gallons of Water; out of the whole was drawn 8 Gallons, and as much Water put in to fill it up; then was drawn out 10 Gallons, after which was put in 6 Gallons of Wine; again there was drawn out

out 5 Gallons; and 7 Gallons of Brandy put in, How much Wine, Brandy, and Water each is, at last in the Mixture?

8. There was a Fish caught whose Head was  $7\frac{1}{2}$  inches long, its Tail was as long as the Head and half, its Body and the Bone was exactly the Length of both Head and Tail, how long was the whole Fish?

9. A Man of Country Fair had a Brid to a String of 25 fine Horses; but not caring to take them at 20 Guineas apiece, the Jockey agreed to let him have them as follows, that was, the first Horse for a single Parthing, the second for an Halfpenny, the third for a Penny, the fourth for Two-pence, and so on, doubling the Price of each Horse to the last; Now I demand the Price of the 25th Horse; and of all the rest added together?

10. There is an Army to which if you add  $\frac{1}{2}$ ,  $\frac{1}{3}$  and  $\frac{1}{4}$  of itself and take away 5000, the Sum Total will be 100000. What is the Number of the whole Army?

11. Suppose a Dog, a Wolf and a Lion, were to devour a Sheep, and that the Dog could eat the Sheep in an Hour, the Wolf in  $\frac{2}{3}$  Hour, and the Lion in  $\frac{1}{2}$  Hour; Now if the Lion began to eat  $\frac{1}{3}$  Hour before the other two, and afterwards all three eat together, the Question is, in what Time the Sheep would be devoured?

12. There are 100 Stones which lie in a Line on the Ground, three Feet asunder, and there is one employed to gather up the Stones one by one, and bring them to a Basket which standeth three Feet from the first Stone. How many Yards must he go backwards and forwards in all, before he hath brought the last Stone to the Basket?

13. Two Boon Companions, Gouty and Pig, are to have (if they can divide it) equal Shares of eight Gallons of Wine, now lying in a Vessel containing exactly eight Gallons: But to make this Partition they shall have only two other empty Vessels, the one containing five, and the other three Gallons; how shall they manage to divide the said Wine?



the  
the  
the  
and  
of  
near  
fol-  
the  
fourth  
each  
25th

$\frac{3}{4}$  0  
0000

your  
Hour  
ow i  
and  
wha

in the  
 red to  
 stone  
 ds in  
 get?  
 are to  
 allon  
 eight  
 om  
 d the  
 le the